- Version: 1.0
- 1 Suppose you are given a magic black box that somehow answers the following decision problem in *polynomial time*:
  - INPUT: A CNF formula  $\varphi$  with n variables  $x_1, x_2, \ldots, x_n$ .
  - OUTPUT: True if there is an assignment of True or False to each variable that satisfies  $\varphi$ .

Using this black box as a subroutine, describe an algorithm that solves the following related search problem in polynomial time:

- INPUT: A CNF formula  $\varphi$  with n variables  $x_1, \ldots, x_n$ .
- OUTPUT: A truth assignment to the variables that satisfies  $\varphi$ , or None if there is no satisfying assignment.

(Hint: You can use the magic box more than once.)

- An *independent set* in a graph G is a subset S of the vertices of G, such that no two vertices in S are connected by an edge in G. Suppose you are given a magic black box that somehow answers the following decision problem in polynomial time:
  - INPUT: An undirected graph G and an integer k.
  - OUTPUT: TRUE if G has an independent set of size k, and False otherwise.
  - **2.A.** Using this black box as a subroutine, describe algorithms that solves the following optimization problem *in polynomial time*:
    - INPUT: An undirected graph G.
    - OUTPUT: The size of the largest independent set in G.

(Hint: You have seen this problem before.)

- **2.B.** Using this black box as a subroutine, describe algorithms that solves the following search problem in polynomial time:
  - INPUT: An undirected graph G.
  - Output: An independent set in G of maximum size.

## To think about later:

Formally, a **proper coloring** of a graph G = (V, E) is a function  $c: V \to \{1, 2, ..., k\}$ , for some integer k, such that  $c(u) \neq c(v)$  for all  $uv \in E$ . Less formally, a valid coloring assigns each vertex of G a color, such that every edge in G has endpoints with different colors. The **chromatic number** of a graph is the minimum number of colors in a proper coloring of G.

Suppose you are given a magic black box that somehow answers the following decision problem in polynomial time:

- INPUT: An undirected graph G and an integer k.
- OUTPUT: TRUE if G has a proper coloring with k colors, and FALSE otherwise.

Using this black box as a subroutine, describe an algorithm that solves the following *coloring problem* in polynomial time:

- INPUT: An undirected graph G.
- OUTPUT: A valid coloring of G using the minimum possible number of colors.

(Hint: You can use the magic box more than once. The input to the magic box is a graph and only a graph, meaning only vertices and edges.)