Prove that each of the following languages is **not** regular.

- **1** $\{0^{2^n} \mid n \ge 0\}.$
- **2** $\{0^{2n}1^n \mid n \ge 0\}$
- **3** $\{0^m 1^n \mid m \neq 2n\}$
- 4 Strings over $\{0,1\}$ where the number of 0s is exactly twice the number of 1s.
- 5 Strings of properly nested parentheses (), brackets [], and braces {}. For example, the string ([]){} is in this language, but the string ([)] is not, because the left and right delimiters don't match.
- 6 Strings of the form $w_1 # w_2 # \cdots # w_n$ for some $n \ge 2$, where each substring w_i is a string in $\{0, 1\}^*$, and some pair of substrings w_i and w_j are equal.

Extra problems

7 $\{w \in (0+1)^* \mid w \text{ is the binary representation of a perfect square}\}$