Designing DFAs via product construction and designing NFAs.
1 Describe a DFA that accepts the following language over the alphabet $\Sigma=\{0,1\}$.
All strings in which the number of 0 s is even and the number of 1 s is not divisible by 3 .
2 All strings that are both the binary representation of an integer divisible by 3 and the ternary (base-3) representation of an integer divisible by 4 .
For example, the string 1100 is an element of this language, because it represents $2^{3}+2^{2}=12$ in binary and $3^{3}+3^{2}=36$ in ternary.
3 Design an NFA for the language $(01)^{+}+(010)^{+}$.

## Work on these later:

Describe deterministic finite-state automata that accept each of the following languages over the alphabet $\Sigma=\{0,1\}$. You may find it easier to describe these DFAs formally than to draw pictures.

4 All strings $w$ such that $\binom{|w|}{2} \bmod 6=4$. (Hint: Maintain both $\binom{|w|}{2} \bmod 6$ and $|w| \bmod 6$.)
5 (Hard.) All strings $w$ such that $F_{\#(10, w)} \bmod 10=4$, where $\#(10, w)$ denotes the number of times 10 appears as a substring of $w$, and $F_{n}$ is the $n$th Fibonacci number:

$$
F_{n}= \begin{cases}0 & \text { if } n=0 \\ 1 & \text { if } n=1 \\ F_{n-1}+F_{n-2} & \text { otherwise }\end{cases}
$$

