CS/ECE 374 A (Spring 2024)
Homework 3 (due Feb 8 Thursday at 10am)

Instructions: As in previous homeworks.

Problem 3.1: For each of the following languages in parts (a), (b), and (c), describe an NFA that accepts the language, using as few states as you can. Provide a short explanation of your solution.

(a) (30 pts) the language defined by the regular expression $((01)^* + (10)^*) \cdot (11 + 00) \cdot \big(\epsilon + (11)^*0 + (00)^*1\big)^*$ over the alphabet $\{0, 1\}$.

(b) (30 pts) all strings $x \in \{0, 1\}^*$ such that (x has 1000 as a substring) and (there exist two distinct indices i's such that ith character of x and the $(i + 3)$rd character of x are different).

[*Note*: if x has 1000 as a substring, we already know that there is at least one such i with the property, but we are asking for at least one more i with the property. For example, 1101001000 is in the language, and so is 0100000100.]

(c) (10 pts) all strings in $\{0, 1\}^*$ that have at least two 1's and for some pair of two consecutive ones, have a block of 0's where the number of 0's is divisible by 3. In other words, the language defined by the regular expression $(0 + 1)^* \cdot 1 \cdot (000)^* \cdot 1 \cdot (0 + 1)^*$.

(d) (30 pts) Convert your NFA from part (c) to a DFA by using the subset construction (i.e., power set construction). [*Note*: Don't include unreachable states; for full points your DFA should not have more than 10 states. It may be possible to collapse some states to reduce the number of states further, but you are not required to do so.]

Problem 3.2: For $\Sigma = \{0, 1, 2\}$, given a string $y \in \Sigma^*$, let y^{IL_0} define a string where 0 is inserted after every character of y. For example if $y = 0121$ then $y^{IL_0} = 00102010$. Given a language L over the alphabet $\Sigma = \{0, 1, 2\}$, define

$$\textsc{Interleave}_0(L) = \{xy^{IL_0}z : \ xyz \in L, \ x, y, z \in \Sigma^*\}.$$

Prove that if L is regular, then $\textsc{Interleave}_0(L)$ is regular.

(For example, if 01001**01**0011 $\in L$, then 01001**0001**00011 $\in \textsc{Interleave}_0(L)$.)

(For a different example: $\textsc{Interleave}_0(1^*) = 1^* \cdot (10)^* \cdot 1^*$.)

[*Hint*: given an NFA (or DFA) for L, construct an NFA for $\textsc{Interleave}_0(L)$. Give a formal description of your construction. Provide an explanation of how your NFA works, including the meaning of each state. A formal proof of correctness of your NFA is not required.]