Instructions: As in previous homeworks.

Problem 2.1: For each of the following languages over the alphabet \( \{0, 1\} \), give a regular expression that describes that language, and briefly argue why your expression is correct.

(a) All strings that begin with 1010101 and have length divisible by 5.
(b) All strings that do not begin with 101 or 010, and end with 00110.
(c) All strings \( x \) such that the number of “leading zeros” in \( x \) is divisible by 3, and \( x \) contains an odd number of ones. (For example, the string 00000101001 has 5 leading zeros and is not in the language; the string 101001 has 0 leading zeros and is in the language.)
(d) All strings that have an even number of occurrences of 01 as a substring. (For example, 11110000 and 0001111011111 are in the language but 000111110001100011 is not.)

Problem 2.2: Describe a DFA that accepts each of the following languages. Describe briefly what each state in your DFA means. For (a)–(b), you should draw the complete DFA. For (c), do not attempt to draw your DFA, since the number of states could be huge; instead, give a mathematically precise description of the states \( Q \), the start state \( s \), the accepting states \( A \), and the transition function \( \delta \).

(a) (25 points) All strings in \( \{0, 1\}^* \) that begin with 1010101 and have an even number of zeros. [Hint: you may either use the product construction or give a direct solution (the latter uses fewer states).]
(b) (30 points) All strings \( x \in \{0, 1\}^* \) that do not begin with 101 or 010, and end with 00110. [Hint: do not use the product construction. Give a direct solution instead. The number of states should be under 15.]
(c) (45 points) All strings \( x \in \{0, 1\}^* \) such that the number of 0’s is divisible by 7 or the number of 1’s is divisible by 11 or the number of occurrences of 01 is even.