CS/ECE 374 A (Spring 2024)
Homework 11 (due Apr 25 Thursday at 10am)

Instructions: As in previous homeworks. See also Past HW 11 for tips and examples of NP-hardness proofs.

Problem 11.1: Consider the following problem called SUPER-SAT:

> *Input*: a CNF Boolean formula F with m clauses and n variables.
>
> *Output*: true iff there exists an assignment such that in each clause, strictly more than half of its literals are true.

(a) (10 pts) Prove that SUPER-SAT is in NP.

(b) (90 pts) Prove that SUPER-SAT is NP-hard, by reduction from the 3SAT problem.
Hint: given a 3CNF formula, create a new 5CNF formula...

Problem 11.2: Consider the following problem called CONNECTIONS:

> *Input*: a set X of M elements, a collection of N subsets $Y_1, \ldots, Y_N \subseteq X$, and numbers A and B with $AB = M$.
>
> *Output*: true iff there exists a partition of X into A disjoint subsets $Z_1, \ldots, Z_A$ each of size exactly B, such that for each $i \in \{1, \ldots, A\}$, the subset Z_i is contained in Y_{j_i} for some $j_i \in \{1, \ldots, N\}$ (the j_i's need not be distinct).

For example: for $X = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, subsets $\{1, 4, 7, 8\}$, $\{2, 3, 5, 7, 9\}$, $\{2, 5, 7, 8\}$, $\{1, 2, 5, 6, 7\}$, and $A = B = 3$, the answer is true (one feasible partition is $\{1, 4, 8\}$, $\{3, 5, 9\}$, $\{2, 6, 7\}$).

[Note: This is inspired by a word puzzle from the New York Times (no, it's not Wordle!). The elements correspond to words, each subset Y_i corresponds to a potential category, and we want to form A groups of B words, such that each group fits in a common category; in the game, $A = B = 4$ and $M = 16$, but the subsets/categories are not given explicitly...]

[For those who don't like games (but who doesn't?), a more serious motivation is in clustering large datasets into A groups...]

(a) (10 pts) Prove that CONNECTIONS is in NP.

(b) (90 pts) Prove that CONNECTIONS is NP-hard, by reduction from the vertex cover problem.
Recall that in the vertex cover (decision) problem, the input is an undirected graph $G = (V, E)$ and a number K, and the output is true iff there exists a subset $S \subseteq V$ of K vertices such that each edge $uv \in E$ has $u \in S$ or $v \in S$.
Hint: let the elements of X correspond to the edges of the given graph; also add some number of extra "dummy" elements. For each vertex v, create a subset containing all edges incident to v together with all the dummy elements...