

Describe deterministic finite-state automata that accept each of the following languages over the alphabet $\Sigma = \{0, 1\}$. You may find it easier to describe these DFAs formally than to draw pictures.

Either drawings or formal descriptions are acceptable, as long as the states Q , the start state s , the accept states A , and the transition function δ are all clear. Try to keep the number of states small.

1. All strings in which the number of **0**s is even **and** the number of **1**s is *not* divisible by 3.
2. All strings in which the number of **0**s is even **or** the number of **1**s is *not* divisible by 3.
3. Given DFAs M_1 and M_2 , all strings in $\overline{L(M_1)} \oplus L(M_2)$.
Recall that for two sets A and B , their symmetric distance $A \oplus B$ is the set of elements in either A or B , but not both.

Work on these later:

4. All strings that are **both** the binary representation of an integer divisible by 3 **and** the ternary (base-3) representation of an integer divisible by 4.

For example, the string **1100** is an element of this language, because it represents $2^3 + 2^2 = 12$ in binary and $3^3 + 3^2 = 36$ in ternary.

5. All strings in which the subsequence **0101** appears an even number of times.
6. All strings w such that $\binom{|w|}{2} \bmod 6 = 4$. [Hint: Maintain both $\binom{|w|}{2} \bmod 6$ and $|w| \bmod 6$.]
- *7. All strings w such that $F_{\#(\mathbf{10}, w)} \bmod 10 = 4$, where $\#(\mathbf{10}, w)$ denotes the number of times **10** appears as a substring of w , and F_n is the n th Fibonacci number:

$$F_n = \begin{cases} 0 & \text{if } n = 0 \\ 1 & \text{if } n = 1 \\ F_{n-1} + F_{n-2} & \text{otherwise} \end{cases}$$