This lab is on reductions, both for undecidability and for NP-Hardness.

1. Prove that $L = \{\langle M \rangle \mid M \text{ accepts a palindrome}\}$ is undecidable.

2. Prove that $L = \{\langle M \rangle \mid M \text{ rejects a palindrome}\}$ is undecidable.

3. Let $L = \{\langle M, w \rangle \mid M \text{ halts on } w \text{ in } |w|^2 \text{ steps}\}$. Prove that L is decidable.

4. Metric Traveling Salesman Tour Problem: given undirected graph $G = (V, E)$ and integer k, is there a closed *walk* of length at most k such that it starts at a vertex s and visits/contains *all* the vertices? A closed walk is also called a tour. Prove that this problem is NP-Hard.

5. Prove that the following problems are NP-hard.
   (a) Given an undirected graph G, is it possible to color the vertices of G with three different colors, so that at most 31337 edges have both endpoints the same color?
   (b) Given an undirected graph G, is it possible to color the vertices of G with three different colors, so that each vertex has at most 8675309 neighbors with the same color?

6. At the end of every semester, Jeff needs to solve the following ExamDesign problem. He has a list of problems, and he knows for each problem which students will *really enjoy* that problem. He needs to choose a subset of problems for the exam such that for each student in the class, the exam includes at least one question that student will really enjoy. On the other hand, he does not want to spend the entire summer grading an exam with dozens of questions, so the exam must also contain as few questions as possible. Prove that the ExamDesign problem is NP-hard. *Hint:* Reduce from Vertex Cover.