MT 1 : Mon Sep 28th 7-9pm

You have 120 minutes to answer five questions.

Write your answers in the separate answer booklet.

Please return this question sheet and your cheat sheet with your answers.

1. For each statement below, check "Yes" if the statement is always true and check "No" otherwise, and give a brief (one short sentence) explanation of your answer. Read these statements very carefully—small details matter!

   (a) Every infinite language is regular.

   (b) The language $(0 + 1(01^*0)^*1)^*$ is not context-free.

   (c) Every subset of an irregular language is irregular.

   (d) The language $\{0^a1^b \mid a - b \text{ is divisible by } 374\}$ is regular.

   (e) If language L is not regular, then L has a finite fooling set.

   (f) If there is a DFA that rejects every string in language L, then L is regular.

   (g) If language L is accepted by an DFA with n states, then its complement $\Sigma^* \setminus L$ is also accepted by a DFA with n states.

   (h) 1^*0^* is a fooling set for the language $\{1^i0^{i+j}1^j \mid i, j \geq 0\}$.

   (i) Every regular language is accepted by a DFA with an odd number of accepting states.

   (j) The context-free grammar

   $$S \to \varepsilon \mid 0A \mid 1S$$
   $$A \to 0S \mid 1A$$

   generates all binary strings with an even number of 0s.

2. Recall that a *run* in a string w is a maximal non-empty substring of w in which all symbols are equal. For any non-empty string $w \in \{0, 1\}^*$, let Delete1st(w) denote the string obtained by deleting the first run in w. For example,

   $$\text{Delete1st}(\underline{111111}) = \varepsilon, \quad \text{Delete1st}(\underline{000}110) = 110, \quad \text{Delete1st}(\underline{1}00110) = 00110.$$

   Let L be an arbitrary regular language over the alphabet $\Sigma = \{0, 1\}$. ***Prove*** that the following languages are also regular.

   (a) $\textsc{Inserted1st}(L) = \{w \in \Sigma^* \mid w \neq \varepsilon \text{ and Delete1st}(w) \in L\}$

   (b) $\textsc{Deleted1st}(L) = \{\text{Delete1st}(w) \mid w \neq \varepsilon \text{ and } w \in L\}$

Take a DFA for L

Build an NFA for new language.

states to track working with first run?

3. For any string $w \in \{0,1\}^*$, let $\mathsf{squish}(w)$ denote the string obtained by dividing w into pairs of symbols, replacing each pair with 0 if the symbols are equal and 1 otherwise, and keeping the last symbol if w has odd length. We can define squish recursively as follows:

$$\mathsf{squish}(w) := \begin{cases} w & \text{if } w = \varepsilon \text{ or } w = 0 \text{ or } w = 1 \\ 0 \cdot \mathsf{squish}(x) & \text{if } w = 00x \text{ or } w = 11x \text{ for some string } x \\ 1 \cdot \mathsf{squish}(x) & \text{if } w = 01x \text{ or } w = 10x \text{ for some string } x \end{cases}$$

For example,

$$\mathsf{squish}(\underbrace{00}\underbrace{10}\underbrace{11}\underbrace{10}\underbrace{01}1) = 010111$$

(a) ***Prove*** that $\#(1, \mathsf{squish}(w)) \le \#(1, w)$ for every string w.

(b) ***Prove*** that $\#(1, \mathsf{squish}(w))$ is even if and only if $\#(1, w)$ is even (or equivalently, that $\#(1, \mathsf{squish}(w)) \bmod 2 = \#(1, w) \bmod 2$) for every string w.

As usual, you can assume any result proved in class, in the lecture notes, in labs, in lab solutions, or in homework solutions. In particular, you may use the fact that $\#(1, xy) = \#(1, x) + \#(1, y)$ for all strings x and y.

strictly

4. Let L be the set of all strings in $\{0,1\}^*$ in which every run of 0s is followed immediately by a *shorter* run of 1s. For example, the strings 0001100000111 and 11100100000111 and 11111 are in L, but the strings 00011111 and 000110000 are not.

   (a) ***Prove*** that L is *not* a regular language. fooling set?

   (b) Describe a context-free grammar for L.

5. For each of the following languages L over the alphabet $\Sigma = \{0,1\}$, describe a DFA that accepts L **and** give a regular expression that represents L. You do **not** need to prove that your answers are correct, but you should give your DFA states mnemonic names.

   (a) Strings that do not contain the subsequence 010. ← after a 0 & a 1, no more 0s

   (b) Strings that contain at least two even-length runs of 1s.

CS/ECE 374 A ✧ Fall 2026

Midterm 1 Practice 1

September 24, 2026

Name:	Emily Fox
NetID:	ekdfox

- ***Don't panic!***
- You have 120 minutes to answer five questions. **The questions are described in more detail in a separate handout.**
- If you brought anything except your writing implements, your **hand-written** double-sided 8½" × 11" cheat sheet, and your university ID, please put it away for the duration of the exam. In particular, please turn off and put away *all* medically unnecessary electronic devices.
- Please clearly print your name and your NetID in the boxes above.
- Please also print your name at the top of every page of the answer booklet, except this cover page. We want to make sure that if a staple falls out, we can reassemble your answer booklet. (It doesn't happen often, but it does happen.)
- Proofs or other justifications beyond those listed in the standard grading rubrics are required if and only if we explicitly ask for them, using the word ***prove*** or ***justify*** in bold italics. **In particular, DFAs need to have mnemonic state names.**

- **Do not write outside the black boxes on each page**. These indicate the area of the page that our scanners will actually scan. If the scanner can't see your work, we can't grade it.
- If you run out of space for an answer, feel free to use the overflow/scratch pages at the back of the answer booklet, but **clearly indicate where we should look**. If we can't find your work, we can't grade it.
- **Only work that is written into the stapled answer booklet will be graded**. We **do not** recommend detaching pages from the answer booklet, because any work on those detached pages will not be graded (and it disrupts our process for uploading solutions to Gradescope.) Please let us know if you detach a page either on purpose or accidentally so we can reattach it.
- We will provide additional blank scratch paper on request, but we will not grade anything written on it.
- **Please return all paper with your answer booklet** including your question sheet, your cheat sheet, and all scratch paper. Please put this loose paper **inside** your answer booklet.

Is between
between is and between
or between between and is
or between and and or
or between or and between
or between between and and
or between or and and
or between or and or?

CS/ECE 374 A ✧ Fall 2026 **Midterm 1 Practice 1 Problem 1**	Name: Emily Fox

For each statement below, check "Yes" if the statement is always true and check "No" otherwise, and give a brief (one short sentence) explanation of your answer. Read these statements very carefully—small details matter!

(a) Every infinite language is regular.

☐ Yes ☒ No — $\{0^n1^n \mid n \geq 0\}$

(b) The language $(0 + 1(01^*0)^*1)^*$ is not context-free.

☐ Yes ☒ No — every reg. language is context-free

(c) Every subset of an irregular language is irregular.

☐ Yes ☒ No — $\emptyset \subseteq \{0^n1^n \mid n \geq 0\}$

(d) The language $\{0^a1^b \mid a - b \text{ is divisible by } 374\}$ is regular.

☒ Yes ☐ No — track $a-b$ mod 374 in a DFA

(e) If language L is not regular, then L has a finite fooling set.

☒ Yes ☐ No — $\emptyset$ is always a fooling & more? ← mean!

(f) If there is a DFA that rejects every string in language L, then L is regular.

☐ Yes ☒ No — reject all, $L = \{0^n1^n\}$

(g) If language L is accepted by an DFA with n states, then its complement $\Sigma^* \setminus L$ is also accepted by a DFA with n states.

☒ Yes ☐ No — $A' = Q \setminus A$

(h) 1^*0^* is a fooling set for the language $\{1^i0^{i+j}1^j \mid i, j \geq 0\}$.

☐ Yes ☒ No — no distinguishing 10 & 1100

(i) Every regular language is accepted by a DFA with an odd number of accepting states.

☒ Yes ☐ No — can add a new unreachable acc. state

(j) The context-free grammar

$$S \rightarrow \varepsilon \mid 0A \mid 1S$$
$$A \rightarrow 0S \mid 1A$$

generates all binary strings with an even number of 0s.

☒ Yes ☐ No — HW 1.2

CS/ECE 374 A ✧ Fall 2026 Midterm 1 Practice 1 Problem 2	Name: Emily Fox

For any non-empty string $w \in \{0,1\}^*$, let Delete1st(w) denote the string obtained by deleting the first run in w. Let L be an arbitrary regular language over the alphabet $\Sigma = \{0,1\}$. ***Prove*** that the following languages are also regular.

(a) $\textsc{Inserted1st}(L) = \left\{ w \in \Sigma^* \;\middle|\; w \neq \varepsilon \text{ and } \mathsf{Delete1st}(w) \in L \right\}$

(b) $\textsc{Deleted1st}(L) = \left\{ \mathsf{Delete1st}(w) \;\middle|\; w \neq \varepsilon \text{ and } w \in L \right\}$

(a) Let $M = (Q, \delta, s, A)$ be a DFA accepting L.

Let $M' = (Q', \delta', s', A')$ be an NFA where

new state

$Q' = \{\bar{s}\} \cup \{burn0, burn1\} \cup Q$

$s' = \bar{s}$ (M' burns off first run before simulating M)

$A' = A$ if $s \notin A$. $A' = A \cup \{burn0, burn1\}$ o.w. $\cancel{000}101$

$\delta'(\bar{s}, 0) = \{burn0\}$

$\delta'(\bar{s}, 1) = \{burn1\}$

$\delta'(burn0, 0) = \{burn0\}$

$\delta'(burn1, 1) = \{burn1\}$

$\delta'(burn0, 1) = \{\delta(s, 1)\}$

$\delta'(burn1, 0) = \{\delta(s, 0)\}$

$\delta'(q, a) = \{\delta(q, a)\}$ $\quad \forall q \in Q, a \in \{0,1\}$

Part (b) on page 6.

CS/ECE 374 A ✧ Fall 2026 Midterm 1 Practice 1 Problem 3	Name: Emily Fox

For any string $w \in \{0,1\}^*$, let squish(w) denote the string obtained by dividing w into pairs of symbols, replacing each pair with 0 if the symbols are equal and 1 otherwise, and keeping the last symbol if w has odd length. (See the question handout for a formal recursive definition.)

(a) ***Prove*** that $\#(1, \text{squish}(w)) \leq \#(1, w)$ for every string w.

(b) ***Prove*** that $\#(1, \text{squish}(w))$ is even if and only if $\#(1, w)$ is even, for every string w.

(a) Let w be an arbitrary string in $\{0,1\}^*$.
Assume for all binary strings x s.t. $|x| < |w|$, $\#(1, \text{squish}(x)) \leq \#(1, w)$.

Suppose $|w| \leq 1$. Then
$\#(1, \text{squish}(w)) = \#(1, w)$ — def. squish ($w = \varepsilon$ or $w = 0$ or $w = 1$)

Suppose $w = 00x$. Then

$$
\begin{aligned}
\#(1, \text{squish}(w)) &= \#(1, \text{squish}(00x)) && w = 00x \\
&= \#(1, 0 \cdot \text{squish}(x)) && \text{def. squish} \\
&= \#(1, \text{squish}(x)) && \text{def. } \# \\
&\leq \#(1, x) && \underline{\text{IH}} \\
&= \#(1, 00x) && \text{def. } \# \times 2 \\
&= \#(1, w) && w = 00x
\end{aligned}
$$

Suppose $w = 11x$. Then

$$
\begin{aligned}
\#(1, \text{squish}(w)) &= \#(1, \text{squish}(11x)) && w = 11x \\
&= \#(1, 0 \cdot \text{squish}(x)) && \text{def. squish} \\
&= \#(1, \text{squish}(x)) && \text{def. } \# \\
&\leq \#(1, x) && \underline{\text{IH}} \\
&= \#(1, 11x) - 2 && \text{def. } \# \\
&< \#(1, 11x) && \text{arith} \\
&= \#(1, w) && w = 11x
\end{aligned}
$$

(could be one line)

cont. page 7

CS/ECE 374 A ✧ Fall 2026 Midterm 1 Practice 1 Problem 4	Name: Emily Fox

Let L be the set of all strings in $\{0,1\}^*$ in which every run of 0s is followed immediately by a *shorter* run of 1s.

strinctly shorter

(a) ***Prove*** that L is *not* a regular language.

(b) Describe a context-free grammar for L.

(a) Let $F = \{0^i \mid i \geq 1\}$.

Let x & y be distinct members of F.

$x = 0^i$ & $y = 0^j$ for some $i \neq j$ & $i, j \geq 1$.

Suppose WLOG $j > i$.

Let $z = 1^{j-1}$

Then $xz = 0^i 1^{j-1} \notin L$, because $j-1 \geq i$.

But $yz = 0^j 1^{j-1} \in L$.

Therefore, F is an infinite fooling set of L & L is not reg.

(b)

$S \to 1S \mid A$ (L)

$A \to \varepsilon \mid BA$ (L but starts with 0)

$B \to 001 \mid 0B \mid 0B1$ (a run of 0s then shorter run of 1s)

CS/ECE 374 A ✧ Fall 2026 Midterm 1 Practice 1 Problem 5	Name: Emily Fox

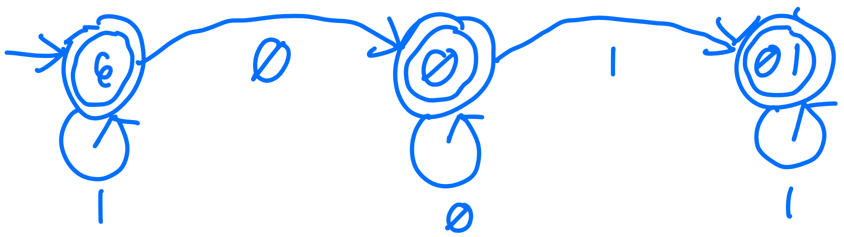
For each of the following languages L over the alphabet $\Sigma = \{0, 1\}$, describe a DFA that accepts L **and** give a regular expression that represents L. You do **not** need to prove that your answers are correct, but you should give your DFA states mnemonic names.

(a) Strings that do not contain the subsequence 010.

(b) Strings that contain at least two even-length runs of 1s.

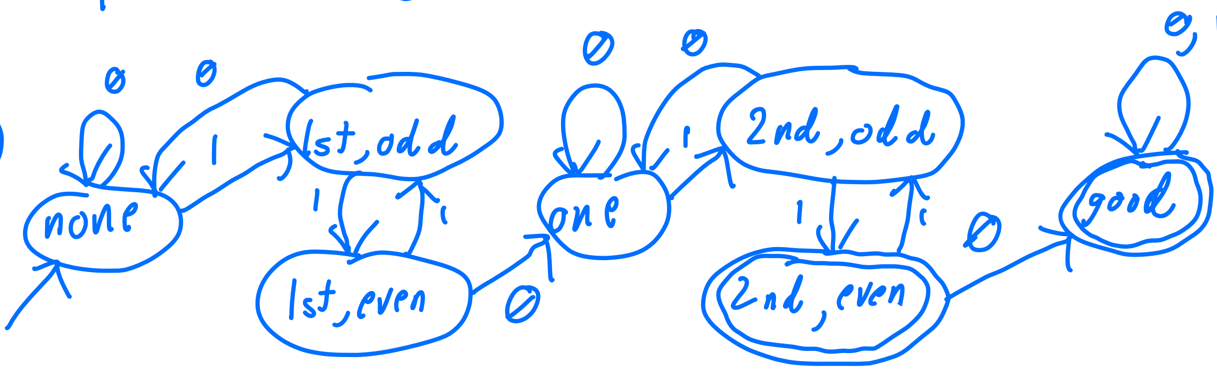
missing transitions go to dump state

(a)



$1^* + 1^*00^* + 1^*00^*11^*$

(b)



$(\varepsilon + (0+1)^*0)11(11)^*(\varepsilon + 0(0+1)^*0)\ 11(11)^*(\varepsilon + 0(0+1)^*)$

2(b): Let $M = (Q, \delta, s, A)$ be a DFA for L.
Construct NFA $M' = (Q', \delta', S', A')$ with multiple start states with ϵ-transitions.

Will simulate a run in M without seeing symbols & then simulate M using symbols we see.

$Q' = Q \times \{run0, run1, see\}$

$S' = \{(\delta(s,0), run0), (\delta(s,1), run1)\}$

$A' = A \times \{run0, run1, see\}$

$\delta'((q, run0), 0) = \emptyset$ $\forall q \in Q$

$\delta'((q, run1), 1) = \emptyset$ $\forall q \in Q$

$\delta'((q, run0), 1) = \{(\delta(q,1), see)\}$

$\delta'((q, see), \epsilon) = \emptyset$

$\delta'((q, run1), 0) = \{(\delta(q,0), see)\}$

$\delta'((q, run0), \epsilon) = \{(\delta(q,0), run0)\}$

$\delta'((q, run1), \epsilon) = \{(\delta(q,1), run1)\}$

$\delta'((q, see), a) = \{(\delta(q,a), see)\}$ $\forall q \in Q, a \in \{0,1\}$

3(a) cont. Suppose $w = abx$ where $a \neq b$. Then,

$$
\begin{aligned}
A(1, squish(w)) &= A(1, squish(abx)) && w = abx \\
&= A(1, 1 \cdot squish(x)) && \text{def. squish}, a \neq b \\
&= 1 + A(1, squish(x)) && \text{def } A \\
&\leq 1 + A(1, x) && \underline{IH} \\
&= A(1, abx) && \text{def } A, a \neq b \\
&= A(1, w) && w = ab
\end{aligned}
$$

In all case $A(1, squish(w)) \leq A(1, w)$.

3(b) Let w be an arbitrary string in $\{0,1\}^*$.
Assume for all strings x s.t. $|x| < |w|$,
$A(1, squish(x)) \bmod 2 = A(1, x) \bmod 2$.

Suppose $|w| \leq 1$. Then,
$A(1, squish(w)) \bmod 2 = A(1, w) \bmod 2$ — def. squish

Suppose $w = abx$ where $a = b$

$$
\begin{aligned}
A(1, squish(w)) \bmod 2 &= A(1, squish(abx)) \bmod 2 && w = abx \\
&= A(1, 0 \cdot squish(x)) \bmod 2 && \text{def. squish}, a = b \\
&= A(1, squish(x)) \bmod 2 && \text{def. } A\,| \\
&= A(1, x) \bmod 2 && \underline{IH} \\
&= A(1, abx) \bmod 2 && \text{def. } A \text{ + mod arith} \\
&= A(1, w) \bmod 2 && abx = w
\end{aligned}
$$

Suppose $w = abx$ where $a \neq b$.

$$
\begin{aligned}
A(1, squish(w)) \bmod 2 &= A(1, squish(abx)) \bmod 2 && w = abx \\
&= A(1, 1 \cdot squish(x)) \bmod 2 && \text{def. squish}, a \neq b \\
&= (1 + A(1, squish(x))) \bmod 2 && \text{def. } A \\
&= (1 + A(1, x)) \bmod 2 && \underline{IH} \text{ + mod. arith.} \\
&= A(1, abx) \bmod 2 && \text{def. } A \\
&= A(1, w) \bmod 2 && w = abx
\end{aligned}
$$

In all cases, $A(1, squish(w)) \bmod 2 = A(1, w) \bmod 2$

(scratch paper)