HW 4 due today 9pm

No GPS or HW next week.

Midterm 1 next Mon Sep 28. 7pm - 9pm

Conflict on Tuesday.

Conflict form or reserve TAC now!

3 practice exams.

Capabilities	Languages	Models
Union Concatenation Repetition	Regular	Regular expressions DFAs NFAs
+ Recursion	Context-free	CFGs - Recursive NFAs - - PDAs -
+ Memory	Decidable (Recursive)	Python programs? Person with a big stack of paper Turing machines

$\{0^n 1^n 0^n \mid n \geq 0\}$

all equivalent

1928: Entscheidungsproblem "decision problem"
is there an algorithm to decide if a given statement has a proof?

Gödel: general recursive functions
Church: (untyped) λ-calculus
Turing: "his machines"

NO!

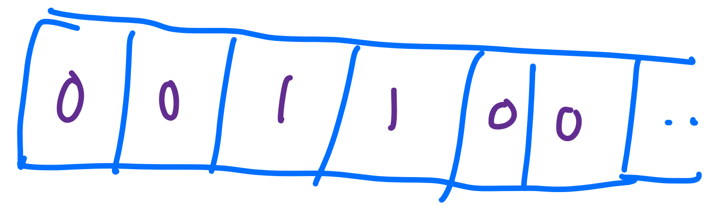
both can do the same thing / model the other

Church-Turing thesis: Both model <u>any</u> finite computation.

Turing machine:

A finite set of states.

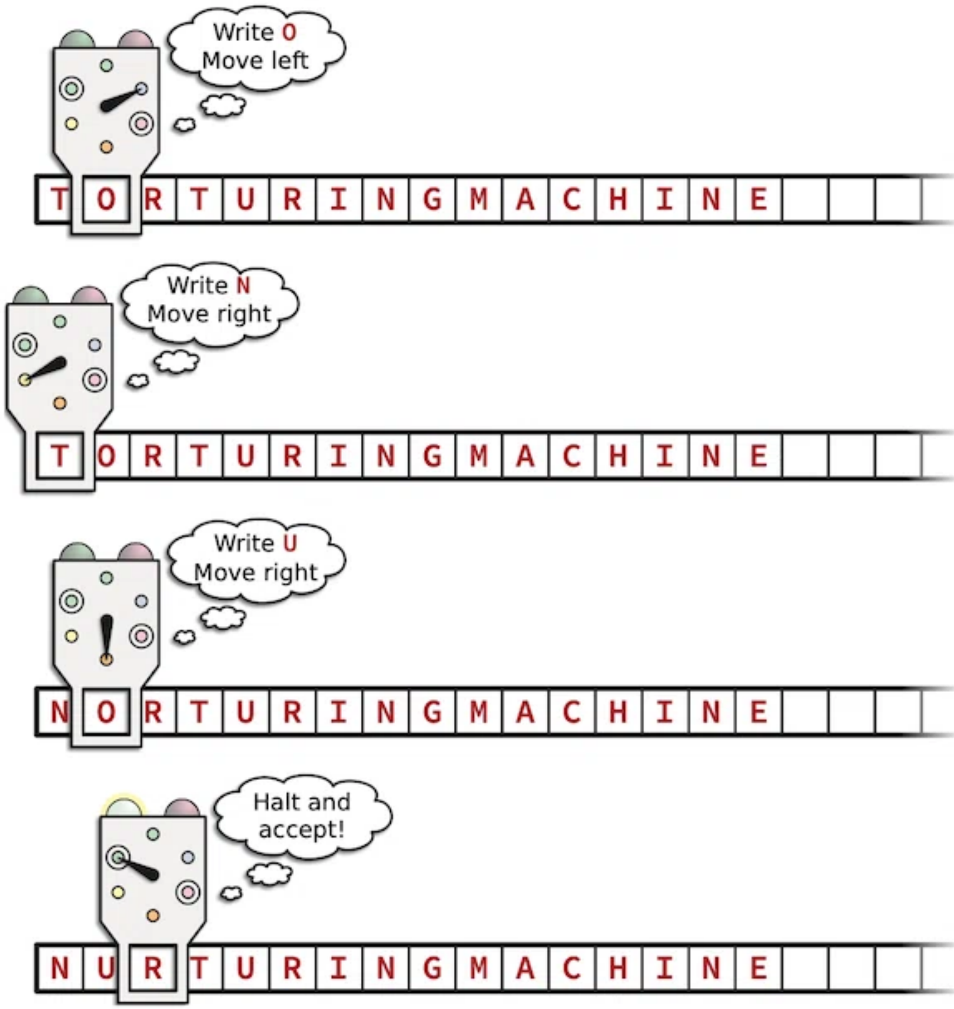
A singly infinite tape containing a sequence of cells.



Initially, the input string w is written on the left side of the tape.

There is a head sitting on one cell.

Machine reads current cell, changes state based current state & cell, writes a new symbol on cell, & move 1 step left or right.



There is a state to accept the string & one to reject the string.

A Turing machine consists of:

- An arbitrary finite set Γ s.t. $|\Gamma| \geq 2$: tape alphabet
- An arbitrary symbol $\square \in \Gamma$: the blank symbol
- An arbitrary nonempty subset $\Sigma \subseteq (\Gamma \setminus \{\square\})$: the input alphabet
- Arbitrary finite set Q: the states
- Three special states (distinct) start, accept, reject $\in Q$

- transition function $\delta : (Q \setminus \{accept, reject\}) \times \Gamma \to Q \times \Gamma \times \{-1, 1\}$

Behavior based on configuration

$(q, x, i) \in Q \times \Gamma^* \times \mathbb{N}$

state

prefix of tape

head's position on tape

0	1	2	3	...

Start in configuration $(start, w, 0)$

input string

Suppose we're in configuration

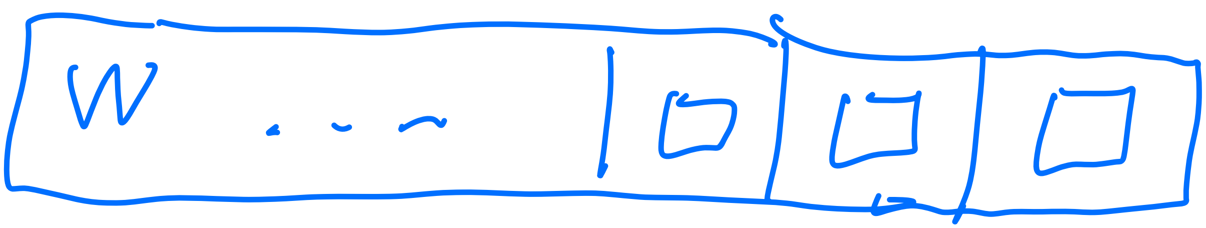
(p, xay, i) where $|x| = i$

If $\delta(p,a) = (q, b, +1)$, go to

configuration $(q, xby, i+1)$.

If $\delta(p,a) = (q, b, -1)$, then go to $(q, xby, i-1)$

Tape beyond w is initially just blanks $\square$



<u>Accept</u> w if state goes accept.

<u>Reject</u> w id state goes to reject.

Crash if going to position -1.

(we'll assume that doesn't happen)

Diverges on w if it never accepts or rejects.

Halts on w if it does not diverge.

Turing machine

$$\text{Accept}(M) := \{ w \mid M \text{ accept } w \}$$

$$\text{Reject}(M) := \{ w \mid M \text{ rejects } w \}$$

$$\text{Accept}(M) \cap \text{Reject}(M) = \emptyset$$

but they aren't necessarily complements!

$$\mathrm{Halt}(M) := \mathrm{Accept}(M) \cup \mathrm{Reject}(M)$$

$$\mathrm{Diverge}(M) := \Sigma^* \setminus \mathrm{Halt}(M)$$

Language L is <u>acceptable</u>, or recursively enumerable, or recognizable, or semi-computable if $L = \mathrm{Accept}(M)$ for some M.

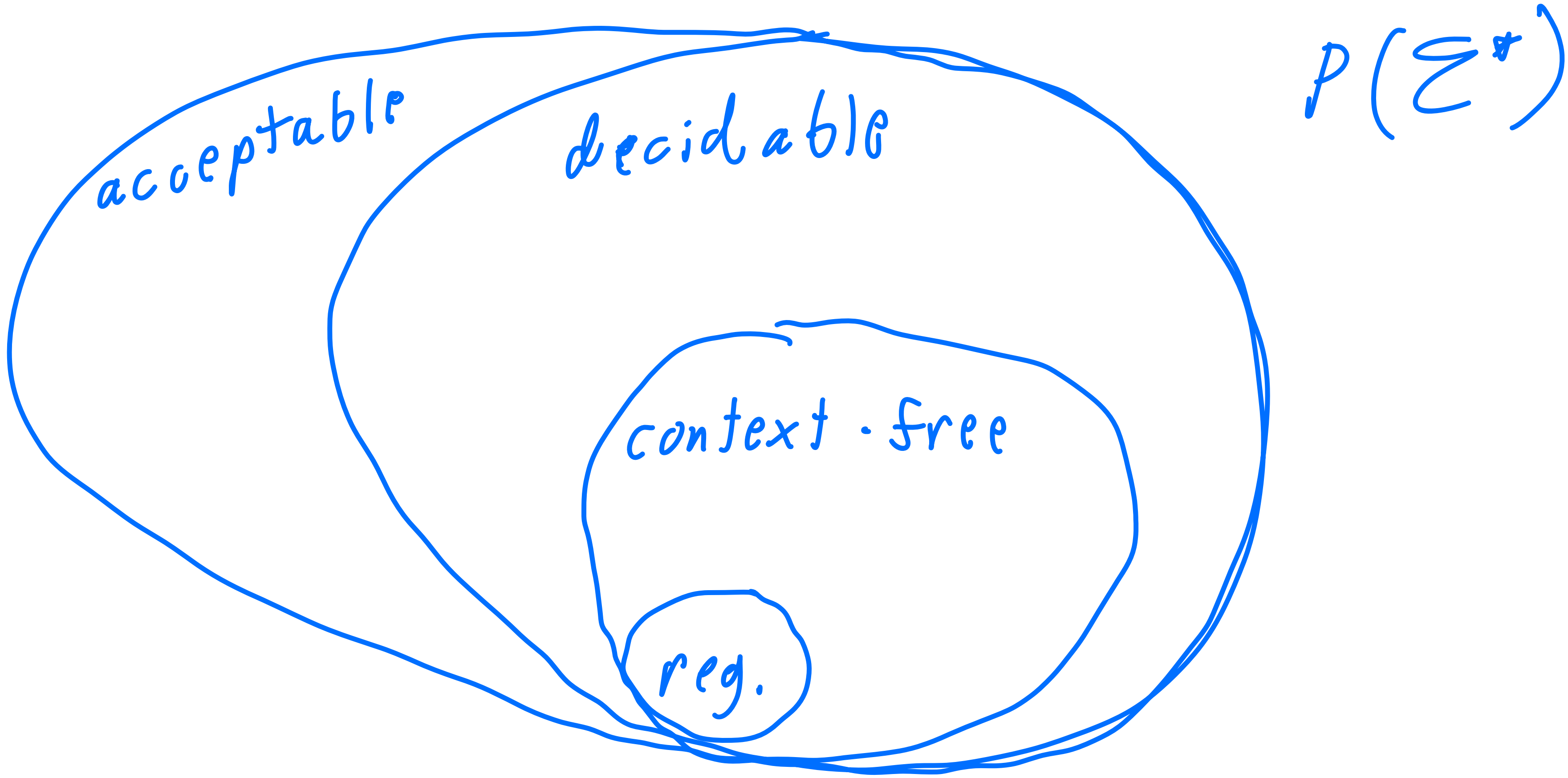
Machine M <u>decides</u> L if (M accepts L & M rejects $\overline{L}$).

i.e. (M accepts L &
M halts on everything.)

L is <u>decidable</u> if there is an M
that decides it.

(or recursive
or computable)

Every decidable language is acceptable
but there are acceptable languages
that are <u>not</u> decidable.



acceptable
decidable
context-free
reg.
P(Σ*)