HW1: Sep 1st (today!)

GPS2: Tue, Sep 8 9pm

HW2: Wed, Sep 9 9pm

Registration: Fri, Sep 4

Want to take a string $w \in \{0,1\}^*$ & check if $\#(1,w) \neq 0 \bmod 3$.

NumIsNotDivisibleBy3($w[1..n]$):

```
rem ← 0
  for i ← 1 to n
      if w[i] = 1
          rem ← rem + 1 (mod 3)
return (rem ≠ 0)
```

answer depends on final state

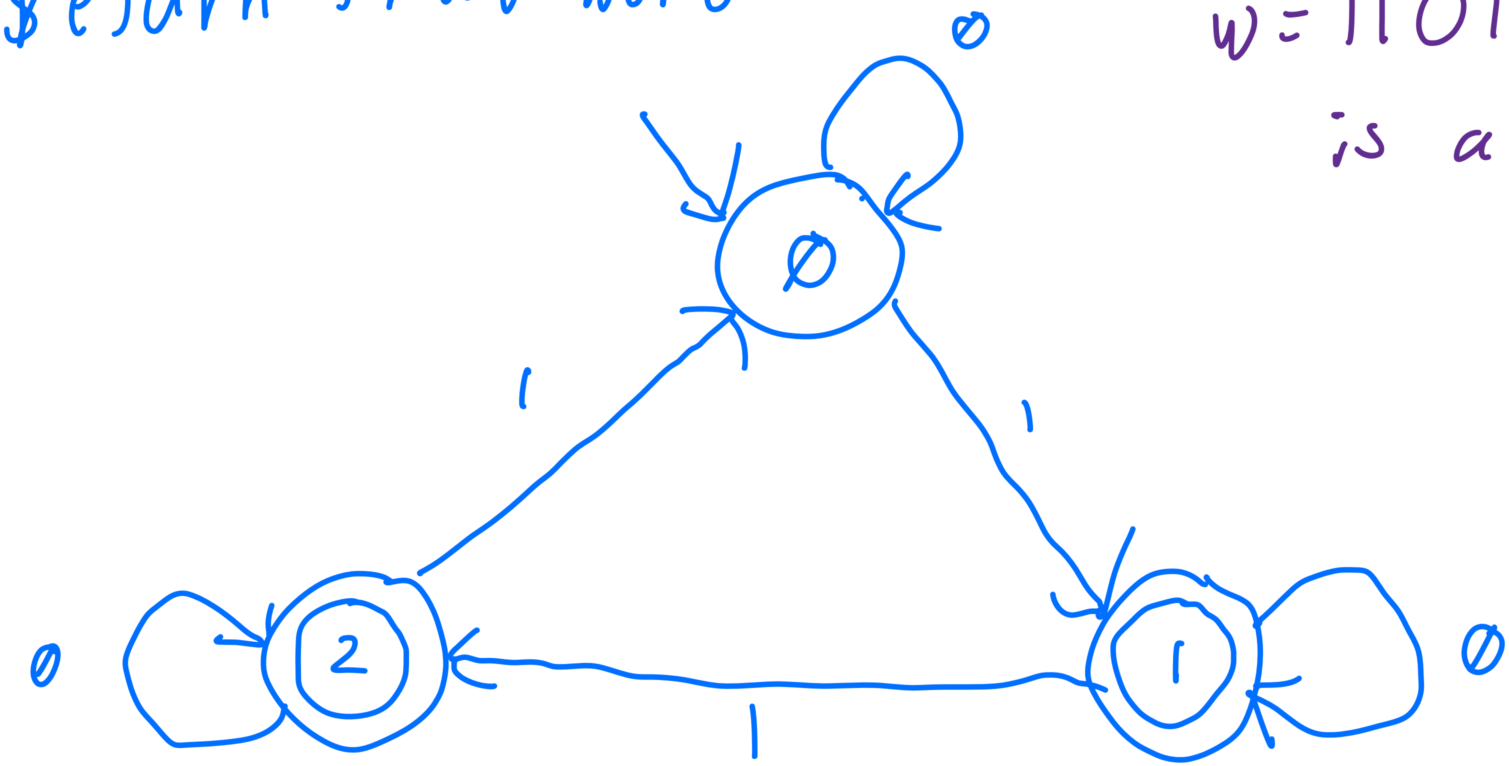
program has 3 states

state changes based on current state + next symbol of w

state/character	0	1
start → 0	0	1
1	1	2
2	2	0

where to go from state 0 reading 1

return true if here



$w = 110101$

is accepted

Finite-state automata (DFA)

Deterministic

finite set of states

state + symbol determins next state

A DFA consists of five components:

- an arbitrary finite set Σ, the input alphabet
- an arbitrary finite set Q called the states
- an arbitrary transition function $\delta : Q \times \Sigma \to Q$
- a start state $s \in Q$
- a subset $A \subseteq Q$ of accept states

Usually denote a machine $M = (\Sigma, Q, \delta, s, A)$.

-or- (Q, δ, s, A) if

Σ clear from context.

Let $w \in \Sigma^*$ be an input string.

M reads one symbol from w at a time, left-to-right (beginning-to-end) begin at state s

It always has a current state q.

It reads symbol a & transitions to state $\delta(q, a)$.

After last symbol, accepts w if current state $q \in A$.

Rejects otherwise.

δ yields an <u>extended transition function</u>

$$\delta^*: Q \times \Sigma^* \to Q$$

$$\delta^*(q, w) := \begin{cases} q & \text{if } w = \varepsilon \\ \delta^*(\delta(q, a), x) & \text{if } w = ax \end{cases}$$

next state

the rest of the way

DFA $M = (Q, \delta, s, A)$ <u>accept</u> input w iff

$$\delta^*(s, w) \in A$$

reject w otherwise

Ex from before $\#(1,w) \not\equiv 0 \bmod 3$.

$\Sigma = \{0,1\}$

$Q = \{0,1,2\}$

$$\delta(q,a) = \begin{cases} q & \text{if } a = 0 \\ (q+1) \bmod 3 & \text{if } a = 1 \end{cases}$$

$s = 0$

$A = \{1,2\}$

$$\begin{aligned} \delta^*(s, 010111) &= \delta^*(\overset{0}{\delta(s,0)}, 10111) \\ &= \delta^*(s, 10111) \\ &= \delta^*(1, 0111) \end{aligned}$$

$\vdots$

$= \delta^*(1, \epsilon)$

$= 1 \in A$

accept!

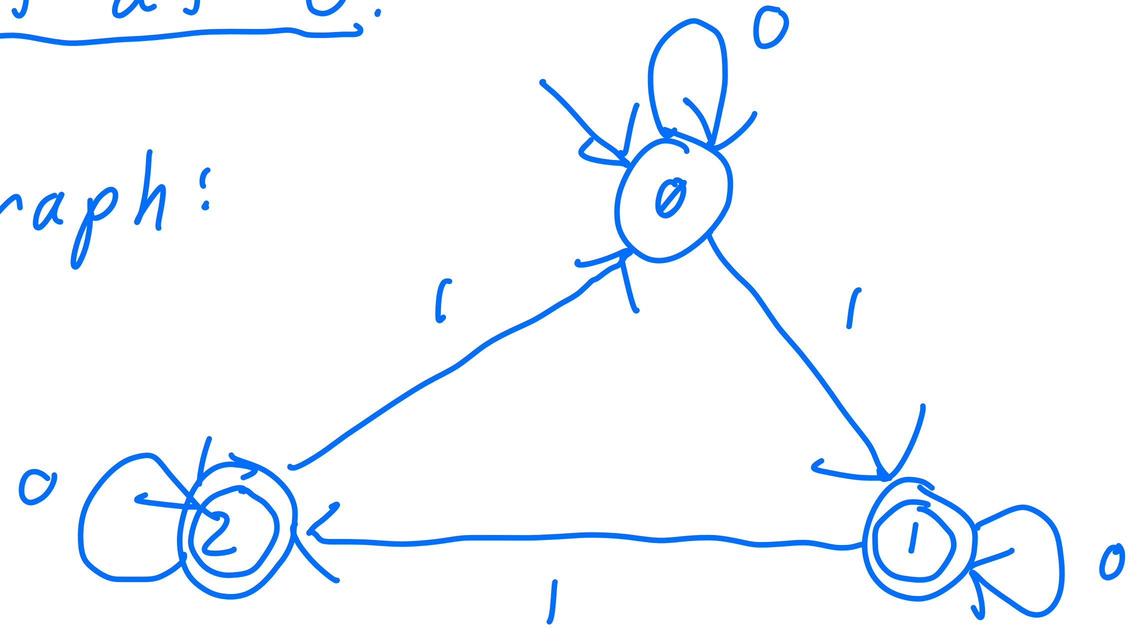
Describing DFAs

- notation like above (good for many states!)
- table

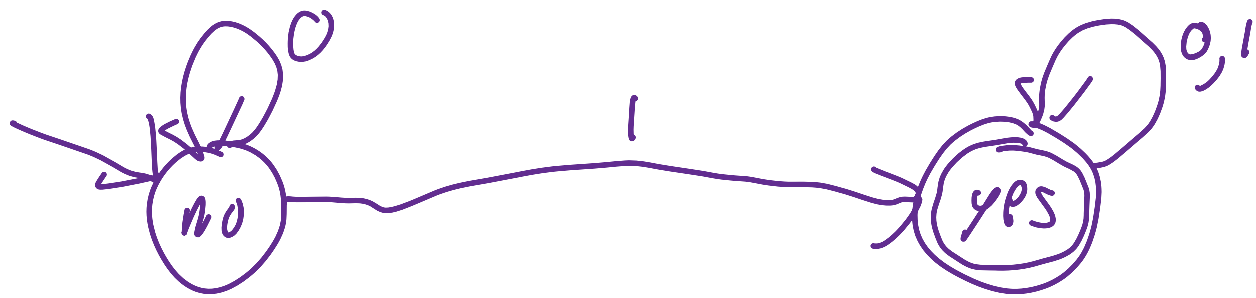
q	$\delta(q,0)$	$\delta(q,1)$	$q \in A$?
0	0	1	no
1	1	2	yes
2	2	0	yes

start at 0.

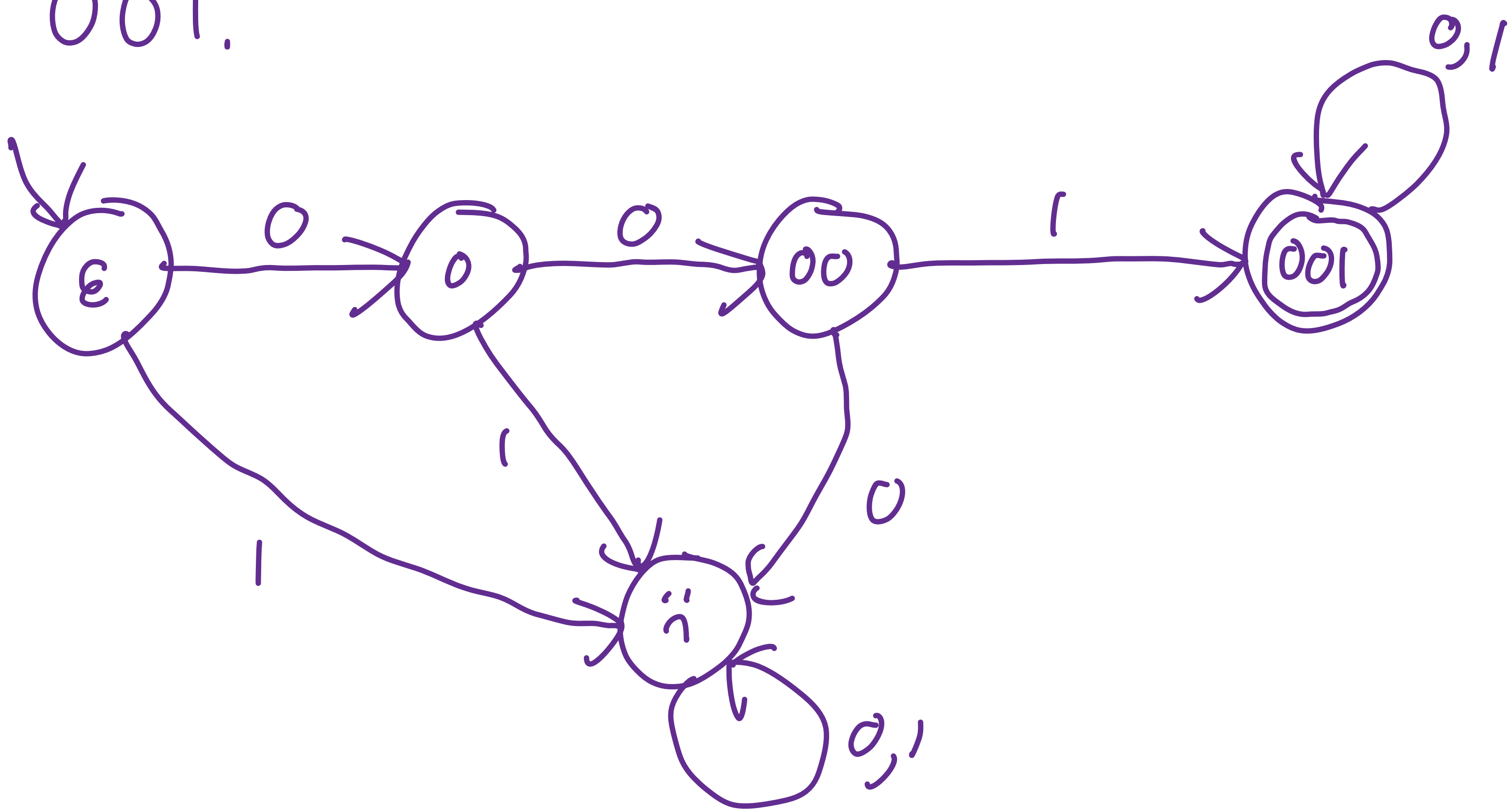
- transition graph:



Ex: Binary strings that contain a 1.



Ex: All binary strings that begin with 001.



Ex: A binary numerals that are a multiple of 5.

let's track "what we've read mod 5"

$Q = \{0, 1, 2, 3, 4\}$

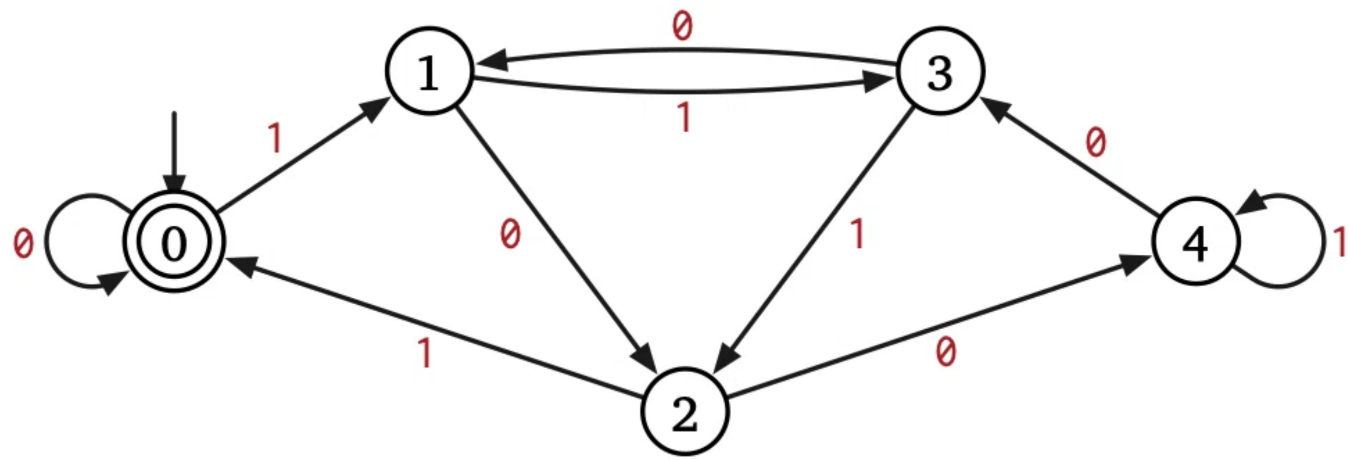
$$\delta(q, a) = \begin{cases} (2q) \bmod 5 & \text{if } a = 0 \\ (2q+1) \bmod 5 & \text{if } a = 1 \end{cases}$$

$s = 0$

$A = \{0\}$

110 ...

110 ...



0
1
2
3
4