

Describe deterministic finite-state automata that accept each of the following languages over the alphabet $\Sigma = \{0, 1\}$. You may find it easier to describe these DFAs formally than to draw pictures.

Either drawings or formal descriptions are acceptable, as long as the states Q , the start state s , the accept states A , and the transition function δ are all clear. Try to keep the number of states small. Or don't. Clarity is more important.

1. All strings in which the number of 0s is even **and** the number of 1s is *not* divisible by 3.
2. All strings in which the number of 0s is even **or** the number of 1s is *not* divisible by 3.
3. All strings that are **both** the binary representation of an integer divisible by 3 **and** the ternary (base-3) representation of an integer divisible by 4.

For example, the string 1100 is an element of this language, because it represents $2^3 + 2^2 = 12$ in binary and $3^3 + 3^2 = 36$ in ternary.

Harder problems to think about later:

4. All strings in which the subsequence 0101 appears an even number of times.
- *5. All strings w such that $\binom{|w|}{2} \bmod 6 = 4$.
[Hint: Maintain both $\binom{|w|}{2} \bmod 6$ and $|w| \bmod 6$.]
[Hint: $\binom{n+1}{2} = \binom{n}{2} + n$.]
- ★6. All strings w such that $F_{\#(10, w)} \bmod 10 = 4$, where $\#(10, w)$ denotes the number of times 10 appears as a substring of w , and F_n is the n th Fibonacci number:

$$F_n = \begin{cases} 0 & \text{if } n = 0 \\ 1 & \text{if } n = 1 \\ F_{n-1} + F_{n-2} & \text{otherwise} \end{cases}$$