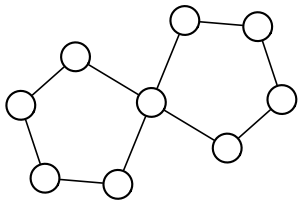
1. FIGURE EIGHT: A ***figure eight of size ℓ*** is an undirected graph with $2\ell-1$ vertices consisting of two cycles C_1 and C_2 of ℓ vertices each sharing exactly one vertex. Prove that it is NP-hard to decide, given an undirected graph $G = (V, E)$ and an integer ℓ, whether G has a figure eight of size at least ℓ as a subgraph.



A figure eight of size 5.

2. HIGH-DEGREE INDEPENDENT SET: Suppose we are given a graph G and an integer k. Prove that it is NP-hard to decide whether G contains an independent set of k vertices, each of which has degree at least k in G.

3. HALF-CLIQUE: Suppose we are given a graph G with $2n$ vertices, for some integer n. Prove that it is NP-hard to decide whether G contains a complete subgraph with n vertices.