1. Recall the following kCOLOR problem: Given an undirected graph G, can its vertices be colored with k colors, so that every edge touches vertices with two different colors?

   (a) Describe a direct polynomial-time many-one reduction from 3COLOR to 4COLOR.

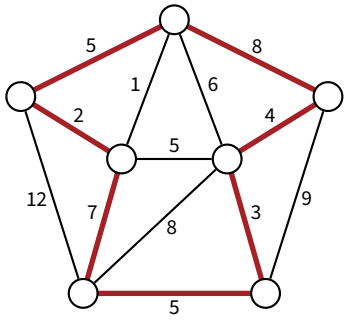
   (b) The 3COLOR problem is NP-hard. Prove that kCOLOR problem is NP-hard for *any* $k \geq 3$.

2. A *Hamiltonian cycle* in a graph G is a cycle that visits every vertex of G exactly once. Deciding whether an arbitrary graph (directed or undirected) contains a Hamiltonian cycle is NP-hard.[1]

   A ***tonian cycle*** in a graph G is a cycle that visits at least *half* of the vertices of G. Prove that deciding whether an undirected graph contains a tonian cycle is NP-hard.

To think about later:

3. Let G be an undirected graph with weighted edges. A Hamiltonian cycle in G is ***heavy*** if the total weight of edges in the cycle is at least half of the total weight of all edges in G. Prove that deciding whether a graph contains a heavy Hamiltonian cycle is NP-hard.



A heavy Hamiltonian cycle. The cycle has total weight 34; the graph has total weight 67.

[1] Similarly, a *Hamiltonian path* in a graph G is a *path* that visits every vertex of G exactly once. Deciding whether an arbitrary graph (directed or undirected) contains a Hamiltonian path is NP-hard.