CS/ECE 374 A ✧ Fall 2025
Conflict Midterm 1 Problem 1 Solution

For each statement below, check "Yes" if the statement is ***always*** true and check "No" otherwise, and give a brief (one short sentence) explanation of your answer. Read these statements very carefully—small details matter!

(a) Every language is either regular or context-free.

[ ] Yes [X] No — There are non-context-free languaes.

(b) Every regular expression that does not contain a Kleene star (or Kleene plus) represents a finite language.

[X] Yes [ ] No — No repetition! A star-free regular expression of length n can only generate strings of length at most n.

(c) For every regular language $L \subseteq \{0,1\}^*$, the language $\{w^C \mid w^R \in L\}$ also regular.

[X] Yes [ ] No — Swap 0- and 1-transitions, swap start and accepting states, and reverse all transitions.

(d) If L has a fooling set of size 374, then every DFA for L has at most 374 states.

[ ] Yes [X] No — Every DFA for L has *at least* 374 states; it could have more.

(e) $(100)^*$ is a fooling set for the language $\{w \in \{0,1\}^* \mid \#(0,w) = \#(1,w)\}$.

[X] Yes [ ] No — The suffix $(011)^i$ distinguishes $(100)^i$ from $(100)^j$.

(f) For any language L and any finite language L', the language $L \cap L'$ is regular.

[X] Yes [ ] No — $L \cap L'$ is finite, and all finite languages are regular

(g) The regular expressions $(00 + 11)^*$ and $(00)^*(11)^*$ represent the same language.

[ ] Yes [X] No — The first language includes the string 1100; the second does not.

(h) The language $\{0^a 0^b \mid a > b\}$ is regular.

[X] Yes [ ] No — This is 00^*; we can always set $b = 0$.

(i) If F is a fooling set for some irregular language, then F^* contains infinitely many strings.

[ ] Yes [X] No — $\varnothing$ is a fooling set for every language, but $\varnothing^*$ contains only one string.

(j) For every context-free language L, the language L^* is also context-free.

[X] Yes [ ] No — $S' \to \varepsilon \mid SS'$

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Conflict Midterm 1 Problem 2 Solution

Recall that a *run* in a string w is a maximal non-empty substring of w in which all symbols are equal. For any non-empty string $w \in \{0, 1\}^*$ let $\text{Delete1st}(w)$ denote the string obtained by deleting the first run in w. For example,

$$\text{Delete1st}(\underline{111}000) = 000, \qquad \text{Delete1st}(\underline{00}1100) = 1100, \qquad \text{Delete1st}(\underline{1}01010) = 01010.$$

Let L be an arbitrary regular language over the alphabet $\Sigma = \{0, 1\}$. ***Prove*** that the following languages are also regular.

(a) $\text{INSERT1ST}(L) = \left\{ w \in \Sigma^* \;\middle|\; w \neq \varepsilon \text{ and } \text{Delete1st}(w) \in L \right\}$

Solution: Let $M = (Q, s, A, \delta)$ be an arbitrary DFA for L. We construct a **DFA** $M' = (Q', S', A', \delta')$ for $\text{INSERT2ND}(L)$ as follows.

$$\begin{aligned} Q' &= Q \times \{\text{start}, \text{first0}, \text{first1}, \textit{after}\} \\ S' &= (s, \text{start}) \\ A' &= A \times \{\text{first0}, \text{first1}, \textit{after}\} \end{aligned}$$

$$\begin{aligned} \delta'((q, \text{start}), 0) &= (q, \text{first0}) \\ \delta'((q, \text{start}), 1) &= (q, \text{first1}) \end{aligned}$$

$$\begin{aligned} \delta'((q, \text{first0}), 0) &= (q, \text{first}) \\ \delta'((q, \text{first0}), 1) &= (\delta(q, 1), \text{after}) \end{aligned}$$

$$\begin{aligned} \delta'((q, \text{first1}), 0) &= (\delta(q, 0), \text{after}) \\ \delta'((q, \text{first1}), 1) &= (1, \text{first1}) \end{aligned}$$

$$\begin{aligned} \delta'((q, \text{after}), 0) &= (\delta(q, 0), \text{after}) \\ \delta'((q, \text{after}), 1) &= (\delta(q, 1), \text{after}) \end{aligned}$$

M' passes the first run in its input string to M, tosses the second run out the window, and then passes the rest of its input string to M. The second component of the state records whether M' is at its start state, M' is reading the first run (which consists of either 0s or 1s), or M' has already read the complete first run. ■

Rubric: 5 points: standard language-transformation rubric (scaled)

(b) $\textsc{Delete1st}(L) = \left\{ \mathsf{Delete1st}(w) \;\middle|\; w \neq \varepsilon \text{ and } w \in L \right\}$

Solution: Let $M = (Q, s, A, \delta)$ be an arbitrary DFA for L. We construct an NFA $M' = (Q', S', A', \delta')$ with ε-transitions for $\textsc{Delete1st}(L)$ as follows.

$$Q' = Q \times \{\mathsf{start}, \mathsf{first0}, \mathsf{first1}, \mathsf{after}\}$$
$$s' = (s, \mathsf{start})$$
$$A' = A \times \{\mathsf{first0}, \mathsf{first1}, \mathsf{after}\}$$

$$\delta'((q, \mathsf{start}), \varepsilon) = \big\{(\delta(q, \mathtt{0}), \mathsf{first0}), (\delta(q, \mathtt{1}), \mathsf{first1})\big\}$$

$$\delta'((q, \mathsf{first0}), \varepsilon) = \big\{(\delta(q, \mathtt{0}), \mathsf{first0})\big\}$$
$$\delta'((q, \mathsf{first0}), \mathtt{1}) = \big\{(\delta(q, \mathtt{1}), \mathsf{after})\big\}$$

$$\delta'((q, \mathsf{first1}), \varepsilon) = \big\{(\delta(q, \mathtt{0}), \mathsf{first1})\big\}$$
$$\delta'((q, \mathsf{first1}), \mathtt{0}) = \big\{(\delta(q, \mathtt{0}), \mathsf{after})\big\}$$

$$\delta'((q, \mathsf{after}), \mathtt{0}) = \big\{(\delta(q, \mathtt{0}), \mathsf{after})\big\}$$
$$\delta'((q, \mathsf{after}), \mathtt{1}) = \big\{(\delta(q, \mathtt{1}), \mathsf{after})\big\}$$

All missing transitions go to the empty set $\varnothing$.

M' guesses an initial run, passes that run to M, aborts if the first symbol in its input string matches the initial run, and otherwise passes its input string to M. The second component of the state records whether M is at uts start state, M is reading the first run (which consists of either 0s or 1s), or M has already read the complete first run. ■

Rubric: 5 points: standard language-transformation rubric (scaled)

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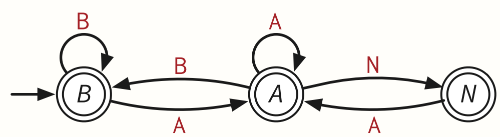
Conflict Midterm 1 Problem 3 Solution

For each of the following languages over the alphabet $\{A, B, N\}$, describe both a regular expression that matches the language and a DFA that accepts the language. You do not need to prove that your answers are correct.

(a) All strings in $\{A, B, N\}^*$ where every N is directly between two As.

Solution:

$$(A+B)^* + (A+B)^* A\,(NA(A+B)^* A)^*\, NA(A+B)^*$$



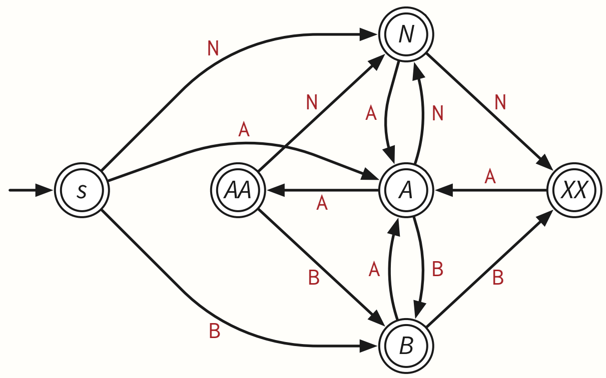
All missing transitions in the DFA go to a hidden dump state. The states are named after the last symbol read, except that *B* is also the start state. ■

Rubric: 5 points = 2½ for regular expression + 2½ for DFA, standard rubrics (scaled). The state labels and explanations are not required for credit.

(b) All strings in which B and N are never adjacent, and every run has length at most 2.

Solution:

$$(\varepsilon + \mathsf{A} + \mathsf{AA})\big((\mathsf{B} + \mathsf{BB} + \mathsf{N} + \mathsf{NN})(\mathsf{A} + \mathsf{AA})\big)^*(\varepsilon\mathsf{B} + \mathsf{BB} + \mathsf{N} + \mathsf{NN})$$



All missing transitions in the DFA go to a hidden dump state. The states are named as follows:

- s: start state
- A: just read one A
- AA: just read two As
- B: just read one B
- NN: just read one N
- XX: just read two Bs or two Ns

■

Rubric: 5 points = 2½ for regular expression + 2½ for DFA, standard rubrics (scaled). The state labels and explanations are not required for credit.

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Conflict Midterm 1 Problem 4 Solution

Prove that $|\text{grow}(\text{grow}(w))| = 2 \cdot \#(0, w) + 3 \cdot \#(1, w)$, for all strings $w \in \{0, 1\}^*$, where the function grow is defined in the question handout.

Solution: Let w be an arbitrary string in Σ^*. Assume for all strings x shorter than w that $|\text{grow}(\text{grow}(x))| = 2 \cdot \#(0, x) + 3 \cdot \#(1, x)$. There are three cases to consider:

- Suppose $w = \varepsilon$. Then we have

$$\begin{aligned}
|\text{grow}(\text{grow}(w))| &= |\text{grow}(\text{grow}(\varepsilon))| && w = \varepsilon \\
&= |\text{grow}(\varepsilon)| && \text{def. grow} \\
&= |\varepsilon| && \text{def. grow} \\
&= 0 && \text{def. } |\cdot| \\
&= 2 \cdot 0 + 3 \cdot 0 && \text{math} \\
&= 2 \cdot \#(0, \varepsilon) + 3 \cdot \#(1, \varepsilon) && \text{def. } \# \\
&= 2 \cdot \#(0, w) + 3 \cdot \#(1, w) && w = \varepsilon
\end{aligned}$$

- Suppose $w = 0x$ for some string x. Then

$$\begin{aligned}
|\text{grow}(\text{grow}(w))| &= |\text{grow}(\text{grow}(0x))| && w = 0x \\
&= |\text{grow}(1 \cdot \text{grow}(x))| && \text{def. grow} \\
&= |01 \bullet \text{grow}(\text{grow}(x))| && \text{def. grow} \\
&= 2 + |\text{grow}(\text{grow}(x))| && \text{def. } |\cdot| \\
&= 2 + 2 \cdot \#(0, x) + 3 \cdot \#(1, x) && \text{ind. hyp.} \\
&= 2(1 + \#(0, x)) + 3 \cdot \#(1, x) && \text{math} \\
&= 2 \cdot \#(0, 0x) + 3 \cdot \#(1, 0x) && \text{def. } \# \\
&= 2 \cdot \#(0, w) + 3 \cdot \#(1, w) && w = 0x
\end{aligned}$$

- Suppose $w = 1x$ for some string x. Then

$$\begin{aligned}
|\text{grow}(\text{grow}(w))| &= |\text{grow}(\text{grow}(1x))| && w = 0x \\
&= |\text{grow}(01 \cdot \text{grow}(x))| && \text{def. grow} \\
&= |101 \bullet \text{grow}(\text{grow}(x))| && \text{def. grow} \\
&= 3 + |\text{grow}(\text{grow}(x))| && \text{def. } |\cdot| \\
&= 3 + 2 \cdot \#(0, x) + 3 \cdot \#(1, x) && \text{ind. hyp.} \\
&= 2 \cdot \#(0, x)) + 3 \cdot (1 + \#(1, x)) && \text{math} \\
&= 2 \cdot \#(0, 1x) + 3 \cdot \#(1, 1x) && \text{def. } \# \\
&= 2 \cdot \#(0, w) + 3 \cdot \#(1, w) && w = 0x
\end{aligned}$$

In all three cases, we conclude that $|\text{grow}(\text{grow}(w))| = 2 \cdot \#(0, w) + 3 \cdot \#(1, w)$. ■

Rubric: 10 points: standard induction rubric. This solution is more detailed than necessary for full credit.

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Conflict Midterm 1 Problem 5 Solution

Let L be the set of all strings $w \in (\mathtt{BAN} + \mathtt{ANA})^*$ in which the substring BAN and ANA appear the same number of times.

(a) ***Prove*** that L is not a regular language.

Solution: Consider the set $F = (\mathtt{BAN})^*$.

Let x and y be distinct strings in F.

Then $x = (\mathtt{BAN})^i$ and $y = (\mathtt{BAN})^j$ for some integers $i \neq j$.

Let $z = (\mathtt{ANA})^i$.

- $xz = (\mathtt{BAN})^i(\mathtt{ANA})^i \in L$, because BAN and ANA each occur exactly i times.
- $yz = (\mathtt{BAN})^j(\mathtt{ANA})^i \notin L$, because BAN occurs j times, ANA occurs i times, and $i \neq j$.

We conclude that F is a fooling set for L.

Because F is infinite, L cannot be regular. ■

Rubric: 5 points: Standard fooling set rubric. This is not the only correct solution.

(b) Describe a context-free grammar for L.

Solution: $S \to \varepsilon \mid \mathtt{BAN}\,S\,\mathtt{ANA}\,S \mid \mathtt{ANA}\,S\,\mathtt{BAN}\,S$

(This is essentially the context-free grammar from homework 1 problem 1.) ■

Solution: $S \to \varepsilon \mid SS \mid \mathtt{BAN}\,S\,\mathtt{ANA} \mid \mathtt{ANA}\,S\,\mathtt{BAN}$ ■

Solution: $S \to \varepsilon \mid ST;\ \ T \to \mathtt{BAN}\,S\,\mathtt{ANA} \mid \mathtt{ANA}\,S\,\mathtt{BAN}$ ■

Rubric: 5 points: Standard CFG rubric. These are not the only correct solutions.