CS/ECE 374 A ✧ Fall 2025
Midterm 1 Problem 1 Solution

For each statement below, check "Yes" if the statement is ***always*** true and check "No" otherwise, and give a brief (one short sentence) explanation of your answer. Read these statements very carefully—small details matter!

(a) For every finite language L, the language L^* is regular.

[X] Yes [ ] No — Finite languages are regular; regular* is regular by definition

(b) For every finite language L, the language L^* is context-free.

[X] Yes [ ] No — All regular languages are context-free.

(c) For every language L, if L^* is regular, then L is regular.

[ ] Yes [X] No — If L is irregular, then so is $L + \mathtt{0} + \mathtt{1}$, but $(L + \mathtt{0} + \mathtt{1})^* = (\mathtt{0} + \mathtt{1})^*$ is regular.

(d) For every regular language L over $\{\mathtt{0}, \mathtt{1}\}$, the language $\{w^C \mid w \in L\}$ also regular.

[X] Yes [ ] No — Swap $\mathtt{0}$- and $\mathtt{1}$-transitions in any DFA for L.

(e) If L has a fooling set of size 374, then every NFA for L requires at least 374 states.

[ ] Yes [X] No — Every *DFA* for L requires at least 374 states, but NFAs can be smaller.

(f) $(\mathtt{01})^*$ is a fooling set for the language $\{w \in \{\mathtt{0}, \mathtt{1}\}^* \mid \#(\mathtt{0}, w) = \#(\mathtt{1}, w)\}$.

[ ] Yes [X] No — No distinguishing suffix for `01` and `0101`.

(g) If F is a fooling set for some regular language L, then F is a regular language.

[X] Yes [ ] No — F must be finite, and all finite languages are regular.

(h) The language $\left\{\mathtt{0}^a \mathtt{0}^b \mid a = b\right\}$ is regular.

[X] Yes [ ] No — This is $(\mathtt{00})^*$.

(i) If language L is accepted by an NFA with n states, then its complement $\Sigma^* \setminus L$ is also accepted by an NFA with n states.

[ ] Yes [X] No — $\{\varepsilon\}$ is accepted by a 1-state NFA, but $\Sigma^* \setminus \{\varepsilon\}$ requires at least 2 states.

(j) For every language L with at least two strings, L^* contains infinite-length strings.

[ ] Yes [X] No — *All* strings have finite length. (The set L^* is infinite, because L contains at least one non-empty string, but each string in L^* is finite.)

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Midterm 1 Problem 2 Solution

For any non-empty string w with length at least 2, let $\text{Delete2nd}(w)$ denote the string obtained by deleting the second symbol in w. Let L be an arbitrary regular language over the alphabet $\Sigma = \{0, 1\}$. ***Prove*** that the following languages are also regular.

(a) $\textsc{Insert2nd}(L) = \left\{ w \in \Sigma^* \;\middle|\; |w| \geq 2 \text{ and } \text{Delete2nd}(w) \in L \right\}$

Solution: Let $M = (Q, s, A, \delta)$ be an arbitrary DFA for L. We construct a **DFA** $M' = (Q', S', A', \delta')$ for $\textsc{Insert2nd}(L)$ as follows.

$$
\begin{aligned}
Q' &= Q \times \{0, 1, \text{many}\} \\
S' &= (s, 0) \\
A' &= \{(q, \text{many}) \mid q \in A\}
\end{aligned}
$$

$$
\begin{aligned}
\delta'((q,0),0) &= (\delta(q,0),1) \\
\delta'((q,0),1) &= (\delta(q,1),1)
\end{aligned}
$$

$$
\begin{aligned}
\delta'((q,1),0) &= (q,\text{many}) \\
\delta'((q,1),1) &= (q,\text{many})
\end{aligned}
$$

$$
\begin{aligned}
\delta'((q,\text{many}),0) &= (\delta(q,0),\text{many}) \\
\delta'((q,\text{many}),1) &= (\delta(q,1),\text{many})
\end{aligned}
$$

State (q, k) in M' means that M is in state q and M' has read k input symbols; many means "more than one". M' passes its first input symbol to M, tosses its second input symbol out the window, and then passes the rest of its input string to M. ■

Rubric: 5 points: standard language-transformation rubric (scaled)

(b) $\text{Delete2nd}(L) = \left\{\text{Delete2nd}(w) \;\middle|\; |w| \geq 2 \text{ and } w \in L\right\}$

Solution: Let $M = (Q, s, A, \delta)$ be an arbitrary DFA for L. We construct an NFA $M' = (Q', S', A', \delta')$ with ε-transitions for $\text{Delete2nd}(L)$ as follows.

$$Q' = Q \times \{0, 1, \text{many}\}$$
$$S' = (s, 0)$$
$$A' = \{(q, \text{many}) \mid q \in A\}$$

$$\delta'((q,0), 0) = \left\{(\delta(q, 0), 1)\right\}$$
$$\delta'((q,0), 1) = \left\{(\delta(q, 1), 1)\right\}$$
$$\delta'((q,0), \varepsilon) = \varnothing$$

$$\delta'((q,1), 0) = \varnothing$$
$$\delta'((q,1), 1) = \varnothing$$
$$\delta'((q,1), \varepsilon) = \left\{(\delta(q, 0), \text{many}),\ (\delta(q, 1), \text{many})\right\}$$

$$\delta'((q,\text{many}), 0) = \left\{(\delta(q, 0), \text{many})\right\}$$
$$\delta'((q,\text{many}), 1) = \left\{(\delta(q, 1), \text{many})\right\}$$
$$\delta'((q,\text{many}), \varepsilon) = \varnothing$$

State (q, k) in M' means that M is in state q and M has read k input symbols; many means “more than one”. M' passes its first input symbol to M, guesses a second symbol to pass to M, and then passes the rest of its input string to M. ■

Rubric: 5 points: standard language-transformation rubric (scaled)

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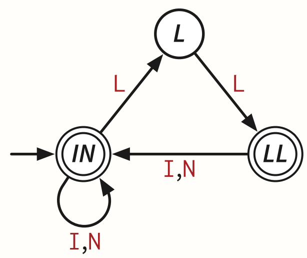
Midterm 1 Problem 3 Solution

For each of the following languages over the alphabet $\{\mathtt{I}, \mathtt{L}, \mathtt{N}\}$, describe both a regular expression that matches the language and a DFA that accepts the language. You do not need to prove that your answers are correct.

(a) All strings in $\{\mathtt{I}, \mathtt{L}, \mathtt{N}\}^*$ where every run of $\mathtt{L}$s has length 2.

Solution:

$$(\mathtt{I}+\mathtt{N})^*\,(\mathtt{LL}(\mathtt{I}+\mathtt{N})(\mathtt{I}+\mathtt{N})^*)^*\,(\mathtt{LL}+\varepsilon)$$



All missing transitions in the DFA go to a hidden dump state. The states are labeled as follows:

- IN: The last symbol read (if any) was not $\mathtt{L}$.
- L: We just read the first $\mathtt{L}$ in a run of $\mathtt{L}$s
- LL: We just read the second $\mathtt{L}$ in a run of $\mathtt{L}$s

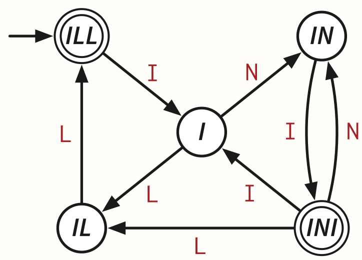
■

Rubric: 5 points = 2½ for regular expression + 2½ for DFA, standard rubrics (scaled). The state labels and explanations are not required for credit.

(b) All strings that are covered by `ILL` and `INI` substrings. A string w is in this language if and only if every character in w is contained in a substring of w that is equal to either `ILL` or `INI`.

Solution:

$$((\mathtt{IN})^*\mathtt{ILL} + (\mathtt{IN})^*\mathtt{INI})^*$$



All missing transitions in the DFA go to a hidden dump state. The states are labeled with the last few symbols read:

- ILL: We haven't read anything, or we just read `ILL`.
- I: We just read `I` (after the start or `ILL` or `INI`)
- IL: We just read `IL`
- IN: We just read `IN`
- INI: We just read `INI`

■

Rubric: 5 points = 2½ for regular expression + 2½ for DFA, standard rubrics (scaled). The state labels and explanations are not required for credit.

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Midterm 1 Problem 4 Solution

Prove that $\#(0, \text{grow}(\text{grow}(w))) \geq \#(0, \text{grow}(w))$ for all strings $w \in \{0, 1\}^*$, where the function grow is defined in the question handout.

Solution: Let w be an arbitrary string in Σ^*. Assume for all strings x shorter than w that $\#(0, \text{grow}(\text{grow}(x))) \geq \#(0, \text{grow}(x))$. There are three cases to consider:

- Suppose $w = \varepsilon$. Then

$$\begin{aligned}
\#(0, \text{grow}(\text{grow}(w))) &= \#(0, \text{grow}(\text{grow}(\varepsilon))) && w = \varepsilon \\
&= \#(0, \text{grow}(\varepsilon)) && \text{Def. grow} \\
&= \#(0, \text{grow}(w)) && w = \varepsilon
\end{aligned}$$

- Suppose $w = 0x$ for some string x. Then

$$\begin{aligned}
\#(0, \text{grow}(\text{grow}(w))) &= \#(0, \text{grow}(\text{grow}(0x))) && w = 0x \\
&= \#(0, \text{grow}(1 \cdot \text{grow}(x))) && \text{def. grow} \\
&= \#(0, 01 \bullet \text{grow}(\text{grow}(x))) && \text{def. grow} \\
&= 1 + \#(0, \text{grow}(\text{grow}(x))) && \text{def. } \# \\
&\geq 1 + \#(0, \text{grow}(x)) && \text{ind. hyp.} \\
&= 1 + \#(0, 1 \cdot \text{grow}(x)) && \text{def. } \# \\
&= 1 + \#(0, \text{grow}(0x)) && \text{def. grow} \\
&= 1 + \#(0, \text{grow}(w)) && w = 0x
\end{aligned}$$

- Suppose $w = 1x$ for some string x. Then

$$\begin{aligned}
\#(0, \text{grow}(\text{grow}(w))) &= \#(0, \text{grow}(\text{grow}(1x))) && w = 0x \\
&= \#(0, \text{grow}(01 \cdot \text{grow}(x))) && \text{def. grow} \\
&= \#(0, 101 \bullet \text{grow}(\text{grow}(x))) && \text{def. grow} \\
&= 1 + \#(0, \text{grow}(\text{grow}(x))) && \text{def. } \# \\
&\geq 1 + \#(0, \text{grow}(x)) && \text{ind. hyp.} \\
&= \#(0, 01 \bullet \text{grow}(x)) && \text{def. } \# \\
&= \#(0, \text{grow}(1x)) && \text{def. grow} \\
&= \#(0, \text{grow}(w)) && w = 0x
\end{aligned}$$

In all three cases, we conclude that $\#(0, \text{grow}(\text{grow}(w))) \geq \#(0, \text{grow}(w))$. ■

Rubric: 10 points: standard induction rubric. This solution is more detailed than necessary for full credit.

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Midterm 1 Problem 5 Solution

Let S be the set of all strings in $(\texttt{ILL} + \texttt{INI})^*$ in which the substrings ILL and INI appear the same number of times.

(a) Prove that S is not a regular language.

Solution: Consider the set $F = (\texttt{ILL})^*$.

Let x and y be distinct strings in F.

Then $x = (\texttt{ILL})^i$ and $y = (\texttt{ILL})^j$ for some integers $i \neq j$.

Let $z = (\texttt{INI})^i$.

- $xz = (\texttt{ILL})^i(\texttt{INI})^i \in S$, because BAN and ANA each occur exactly i times.
- $yz = (\texttt{ILL})^j(\texttt{INI})^i \notin S$, because BAN occurs j times, ANA occurs i times, and $i \neq j$.

We conclude that F is a fooling set for S.

Because F is infinite, S cannot be regular. ■

Rubric: 5 points: Standard fooling set rubric. This is not the only correct solution.

(b) Describe a context-free grammar for S.

Solution: $S \to \varepsilon \mid \texttt{ILL}\,S\,\texttt{INI}\,S \mid \texttt{INI}\,S\,\texttt{ILL}\,S$

(This is essentially the context-free grammar from homework 1 problem 1.) ■

Solution: $S \to \varepsilon \mid SS \mid \texttt{ILL}\,S\,\texttt{INI} \mid \texttt{INI}\,S\,\texttt{ILL}$ ■

Solution: $S \to \varepsilon \mid ST; \quad T \to \texttt{ILL}\,S\,\texttt{INI} \mid \texttt{INI}\,S\,\texttt{ILL}$ ■

Rubric: 5 points: Standard CFG rubric. These are not the only correct solutions.