You have 120 minutes to answer five questions.

Write your answers in the separate answer booklet.

Please return this question sheet and your cheat sheet with your answers.

1. For each statement below, check "Yes" if the statement is always true and check "No" otherwise, and give a brief (one short sentence) explanation of your answer. Read these statements very carefully—small details matter!

   (a) Every integer in the empty set is prime. Yes

   (b) The language $\{0^m 1^n \mid m+n \leq 374\}$ is regular.

   (c) The language $\{0^m 1^n \mid m-n \leq 374\}$ is regular.

   At most one

   (d) For all languages L, the language L^* is regular.

   (e) For all languages L, the language L^* is infinite.

   (f) For all languages $L \subseteq \Sigma^*$, if L can be represented by a regular expression, then $\Sigma^* \setminus L$ is recognized by a DFA. Yes

   (g) For all languages L and L', if $L \cap L' = \varnothing$ and L' is not regular, then L is regular.

   L L'

   good bad

   At most one

   (h) Every regular language is recognized by a DFA with at least 374 accepting states.

   (i) Every regular language is recognized by an NFA with at most 374 accepting states.

   (j) Every context-free language has an infinite fooling set.

2. The *parity* of a bit-string w is 0 if w has an even number of 1s, and 1 if w has an odd number of 1s. For example:

$$parity(\varepsilon) = 0 \qquad parity(0010100) = 0 \qquad parity(00101110100) = 1$$

   (a) Give a *self-contained*, formal, recursive definition of the *parity* function. (In particular, do ***not*** refer to # or other functions defined in class.)

   transform

   (b) ***Prove*** that for every regular language L, the language $\textsc{OddParity}(L) := \{w \in L \mid parity(w) = 1\}$ is also regular.

   (c) ***Prove*** that for every regular language L, the language $\textsc{AddParity}(L) := \{parity(w) \cdot w \mid w \in L\}$ is also regular.

   $011 \to \underline{1}011$

3. For each of the following languages over the alphabet $\Sigma = \{0, 1\}$, either ***prove*** that the language is regular or ***prove*** that the language is not regular. Both of these languages contain the string 00110100000110100.

   (a) $\{0^n w 0^n \mid w \in \Sigma^+ \text{ and } n > 0\}$

   (b) $\{w 0^n w \mid w \in \Sigma^+ \text{ and } n > 0\}$

   Fooling / RegEx

   [Hint: Exactly one of these two languages is regular.]

4. For any string $w \in \{0, 1\}^*$, let take2skip2(w) denote the subsequence of w containing symbols at positions $1, 2, 5, 6, 9, 10, \ldots 4i+1, 4i+2, \ldots$. In other words, take2skip2(w) takes the first two symbols of w, skips the next two, takes the next two, skips the next two, and so on.For example:

   $$\text{take2skip2}(\underline{1}) = 1$$
   $$\text{take2skip2}(\underline{01}0) = 01$$
   $$\text{take2skip2}(\underline{01}00\underline{11}11\underline{00}01\underline{1}) = 0111001$$

   Let L be an arbitrary regular language over $\{0, 1\}$.

   (a) ***Prove*** that the language $\{w \in \Sigma^* \mid \text{take2skip2}(w) \in L\}$ is regular.

   (b) ***Prove*** that the language $\{\text{take2skip2}(w) \mid w \in L\}$ is regular.

) Transformation

5. For each of the following languages L over the alphabet $\{0, 1\}$, describe a DFA that accepts L **and** give a regular expression that represents L. You do ***not*** need to prove that your answers are correct.

   (a) All strings in which every run of 1s has even length and every run of 0s has odd length. (Recall that a *run* is a maximal substring in which all symbols are equal.)

   (b) All strings in 0^*10^* whose length is a multiple of 3.

CS/ECE 374 A ✧ Fall 2025

Midterm 1 Practice 3

September 28, 2025

Name:	Jeff E
NetID:	jeffe

- ***Don't panic!***
- You have 120 minutes to answer five questions. The questions are described in more detail in a separate handout.
- If you brought anything except your writing implements, your **hand-written** double-sided 8½" × 11" cheat sheet, and your university ID, please put it away for the duration of the exam. In particular, please turn off and put away *all* medically unnecessary electronic devices.
- Please clearly print your name and your NetID in the boxes above.
- Please also print your name at the top of every page of the answer booklet, except this cover page. We want to make sure that if a staple falls out, we can reassemble your answer booklet. (It doesn't happen often, but it does happen.)
- Proofs or other justifications are required for full credit if and only if we explicitly ask for them, using the word ***prove*** or ***justify*** in bold italics.
- **Do not write outside the black boxes on each page**. These indicate the area of the page that our scanners will actually scan. If the scanner can't see your work, we can't grade it.
- If you run out of space for an answer, please use the overflow/scratch pages at the back of the answer booklet, but **please clearly indicate where we should look**. If we can't find your work, we can't grade it.
- **Only work that is written into the stapled answer booklet will be graded**. In particular, you are welcome to detach scratch pages from the answer booklet, but any work on those detached pages will not be graded. Please let us know if you detach a page accidentally. We will provide additional scratch paper on request.
- Please return ***all*** paper with your answer booklet: your question sheet, your cheat sheet, and all scratch paper. **Please put all loose paper *inside* your answer booklet.**

Is between
between is and between
or between between and is
or between and and or
or between or and between
or between between and and
or between or and and
or between or and or?

CS/ECE 374 A ✧ Fall 2025 **Midterm 1 Practice 2 Problem 1**	Name: JeffE

For each statement below, check "Yes" if the statement is always true and check "No" otherwise, and give a brief (one short sentence) explanation of your answer. Read these statements very carefully—small details matter!

(a) Every integer in the empty set is ~~prime~~.

[X] Yes [ ] No   Vacuously

(b) The language $\{0^m 1^n \mid m + n \le 374\}$ is regular.

[X] Yes [ ] No   Finite!

(c) The language $\{0^m 1^n \mid m - n \le 374\}$ is regular.

[ ] Yes [X] No $F = \{0^m \mid m > 374\}$ 0^i and 0^j are distinguished by 1^{j-374} where $i < j$

(d) For all languages L, the language L^* is regular. $L = \{0^{n^2} 1 \mid n \ge 0\}$

[ ] Yes [X] No $L = \{0^n 1^n \mid n \ge 0\}$ $L^* = \{0^a 1^a 0^b 1^b 0^c 1^c \cdots 0^z 1^z \mid a, b, c \ldots z \ge 0\}$

(e) For all languages L, the language L^* is infinite.

[ ] Yes [X] No $\emptyset^* = \{\varepsilon\} = \{\varepsilon\}^*$

(f) For all languages $L \subseteq \Sigma^*$, if L can be represented by a regular expression, then $\Sigma^* \setminus L$ is recognized by a DFA.

[X] Yes [ ] No   regex for L $\Leftrightarrow$ DFA for L $\leftrightarrow$ DFA for $\Sigma^* \setminus L$ $A' = Q \setminus A$

(g) For all languages L and L', if $L \cap L' = \emptyset$ and L' is not regular, then L is regular.

[ ] Yes [X] No $L = \{0^n 1^n \mid n \ge 1\}$ $L' = \{1^n 0^n \mid n \ge 1\}$

(h) Every regular language is recognized by a DFA with at least 374 accepting states.

[X] Yes [ ] No   Add 374 isolated accepting states to any DFA   0,1   × 374

(i) Every regular language is recognized by an NFA with at most 374 accepting states.

[X] Yes [ ] No   Thompson's algo → NFA with one accepting state

(j) Every context-free language has an infinite fooling set.

[ ] Yes [X] No $\emptyset$ is context-free $S \to S$

Σ^* is context free $S \to 0S \mid 1S \mid \varepsilon$

CS/ECE 374 A ✧ Fall 2025 Midterm 1 Practice 2 Problem 2	Name:

The *parity* of a bit-string w is `0` if w has an even number of `1`s, and `1` if w has an odd number of `1`s.

(a) Give a *self-contained*, formal, recursive definition of the *parity* function. (In particular, do ***not*** refer to # or other functions defined in class.)

(b) ***Prove*** that for every regular language L, the language $\textsc{OddParity}(L) := \{w \in L \mid parity(w) = 1\}$ is also regular.

(c) ***Prove*** that for every regular language L, the language $\textsc{AddParity}(L) := \{\underline{parity(w)} \cdot w \mid w \in L\}$ is also regular.

even parity!!

(a)

$$parity(w) = \begin{cases} 0 & w = \varepsilon \\ parity(x) & w = 0x \text{ for some string } x \\ 1 - parity(x) & w = 1x \quad \text{" " " "} \end{cases}$$

$\neg parity(x)$

$!\,parity(x)$

(b) $\text{OddParity}(L) = L \cap \text{OddParity}(\Sigma^*)$

$= L \cap 0^*1(0^*10^*1)^*0^*$ ✓

OP(L) is intersection of two reg. langs!

(c) $\text{AddParity}(L) = (1+0)L \cap \text{EvenParity}(\Sigma^*)$

$= (1+0)L \cap (0^*10^*1)^*0^*$

AP(L) is intersection of two reg langs ✓

CS/ECE 374 A ✧ Fall 2025 Midterm 1 Practice 2 Problem 3	Name:

For each of the following languages over the alphabet $\Sigma = \{0, 1\}$, either ***prove*** that the language is regular or ***prove*** that the language is not regular. Both of these languages contain the string $\underbrace{0011010}_{w}0000\underbrace{110100}_{w}$.

(a) $\{0^n w 0^n \mid w \in \Sigma^+ \text{ and } n > 0\}$

(b) $\{w 0^n w \mid w \in \Sigma^+ \text{ and } n > 0\}$

(a) $0^n w 0^n = 0 \underbrace{x 0}$ $\qquad x = 0^{n-1} w 0^{n-1}$

$\boxed{\text{Regular! } 0(0+1)^*0}$

(b) Not regular

Focus on

$\{1^m 0^n 1^m \mid n, m \geq 1\} = L'$

$L \cap 1^*0^*1^*$

Let $F = 11^*$

Let $x \neq y$ be any strings in F

Then $x = 1^a$ $\quad y = 1^b$ for some $a \neq b$

Let $z = 01^a$

Then $xz = 1^a 0 1^a = w 0^n w$ where $w = 1^a$ $n = 1$

$\in L$

But $yz = 1^b 0 1^a$

If $yz = w 0^n w$ then $n = 1$

$w = 1^b$ and $w = 1^a$

impossible because $a \neq b$

So F is ∞ fooling set!

CS/ECE 374 A ✧ Fall 2025 Midterm 1 Practice 2 Problem 4	Name:

For any string $w \in \{0,1\}^*$, let take2skip2(w) denote the subsequence of w containing symbols at positions $1, 2, 5, 6, 9, 10, \ldots 4i+1, 4i+2, \ldots$. In other words, take2skip2(w) takes the first two symbols of w, skips the next two, takes the next two, skips the next two, and so on. Let L be an arbitrary regular language over $\{0,1\}$.

(a) ***Prove*** that the language $\{w \in \Sigma^* \mid \text{take2skip2}(w) \in L\}$ is regular.

(b) ***Prove*** that the language $\{\text{take2skip2}(w) \mid w \in L\}$ is regular.

(a) Let $M = (Q, s, A, \delta)$ be any DFA for L

We build DFA $M' = (Q', s', A', \delta')$ as follows:

$Q' = Q \times \{0,1,2,3\}$ ← #input symbols read by M' mod 4

$s' = (s, 0)$

$A' = A \times \{0,1,2,3\}$

$\delta'((q,0),a) = (\delta(q,a), 1)$

$\delta'((q,1),a) = (\delta(q,a), 2)$

$\delta'((q,2),a) = (q, 3)$

$\delta'((q,3),a) = (q, 0)$

(b) We build NFA with ε-transitions $M' = (Q', s', A', \delta')$ as follows:

$Q' = Q \times \{0,1,2,3\}$ ← #symbols read by $\underline{M}$ mod 4

$s' = (s, 0)$

$A' = A \times \{0,1,2,3\}$

$\delta'((q,0),a) = \{(\delta(q,a), 1)\}$

$\delta'((q,1),a) = \{(\delta(q,a), 2)\}$

$\delta'((q,2),a) = \emptyset$

$\delta'((q,3),a) = \emptyset$

$\delta'((q,0),\varepsilon) = \emptyset$

$\delta'((q,1),\varepsilon) = \emptyset$

$\delta'((q,2),\varepsilon) = \{(\delta(q,1),3), (\delta(q,0),3)\}$

$\delta'((q,3),\varepsilon) = \{(\delta(q,0),0), (\delta(q,1),0)\}$

CS/ECE 374 A ✧ Fall 2025 Midterm 1 Practice 2 Problem 5	Name:

For each of the following languages L over the alphabet $\Sigma = \{0, 1\}$, describe a DFA that accepts L **and** give a regular expression that represents L. You do ***not*** need to prove that your answers are correct.

(a) All strings in which every run of 1s has even length and every run of 0s has odd length.

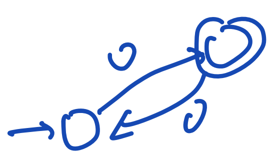
(b) All strings in 0^*10^* whose length is a multiple of 3.

---

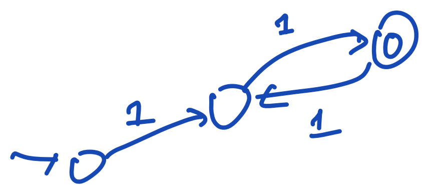
a $(\varepsilon + \text{run1})(\text{run0} \cdot \text{run1})^*(\varepsilon + \text{run0})$

↓

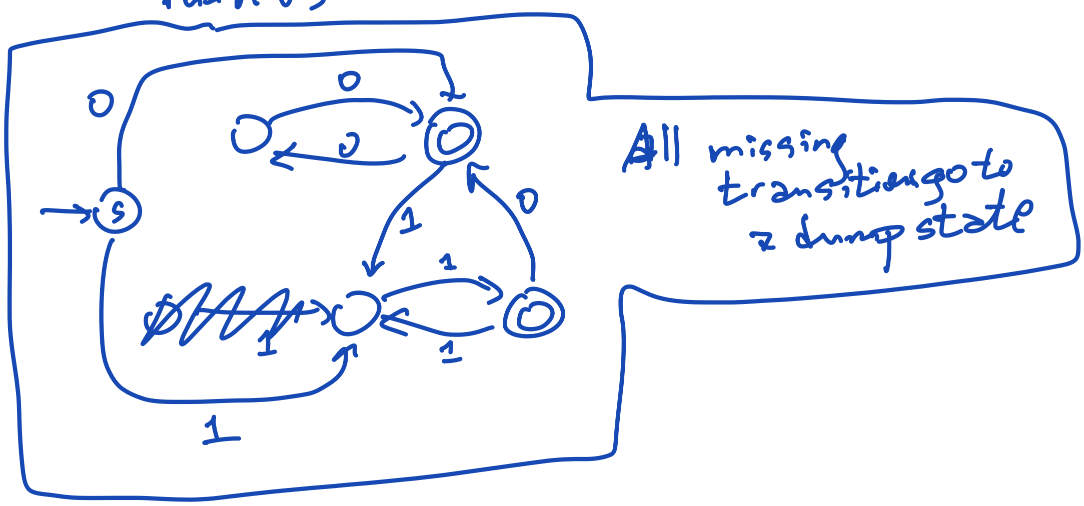
$(\varepsilon + 11(11)^*)(0(00)^* \cdot 11(11)^*)^*(\varepsilon + 0(00)^*)$



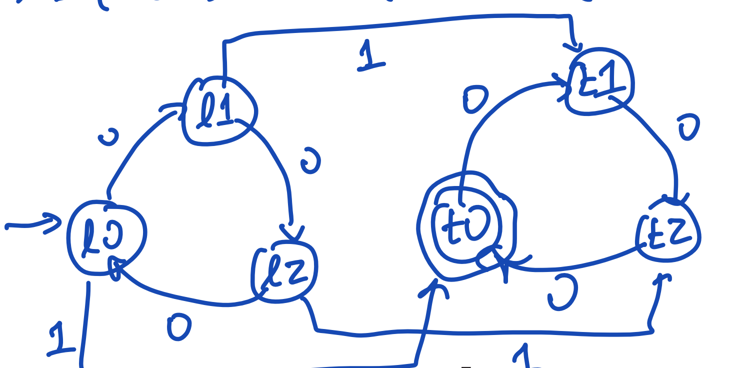
odd-length run of 0's



even-length run of 1's



b $(000)^*1(000)^* + 0(000)^*1(000)^*00 + 00(000)^*1(000)^*0$



All missing transitions go to dump state.

(scratch paper)

(scratch paper)

(scratch paper)