You have 120 minutes to answer five questions.

Write your answers in the separate answer booklet.

Please return this question sheet and your cheat sheet with your answers.

1. For each statement below, check "Yes" if the statement is always true and check "No" otherwise, and give a brief (one short sentence) explanation of your answer. Read these statements very carefully—small details matter!

   (a) Every infinite language is regular.

   (b) The language $(0 + 1(01^*0)^*1)^*$ is not context-free.

   (c) Every subset of an irregular language is irregular.

   (d) The language $\{0^a1^b \mid a - b \text{ is divisible by } 374\}$ is regular.

   (e) If language L is not regular, then L has a finite fooling set.

   (f) If there is a DFA that rejects every string in language L, then L is regular.

   (g) If language L is accepted by an DFA with n states, then its complement $\Sigma^* \setminus L$ is also accepted by a DFA with n states.

   (h) 1^*0^* is a fooling set for the language $\{1^i0^{i+j}1^j \mid i, j \geq 0\}$.

   (i) Every regular language is accepted by a DFA with an odd number of accepting states.

   (j) The context-free grammar $S \to \varepsilon \mid 0S1S \mid 1S0S$ generates all strings in which the number of 0s equals the number of 1s. HW1? Yes

2. For any string w, let cycleleft(w) denote the string obtained by moving the first symbol of w (if any) to the end. More formally:

$$\text{cycleleft}(w) = \begin{cases} \varepsilon & \text{if } w = \varepsilon \\ x \bullet a & \text{if } w = ax \text{ for some symbol } a \text{ and string } x \end{cases}$$

   For example, cycleleft$(001111) = 011110$.

   Let L be an arbitrary regular language over the alphabet $\{0, 1\}$.

   (a) Prove that $\textsc{CycleLeft}(L) = \{\text{cycleleft}(w) \mid w \in L\}$ is a regular language.

   (b) Prove that $\textsc{CycleRight}(L) = \{w \in \Sigma^* \mid \text{cycleleft}(w) \in L\}$ is a regular language.

Transformation

3. For any string $w \in \{0, 1\}^*$, let squish(w) denote the string obtained by dividing w into pairs of symbols, replacing each pair with 0 if the symbols are equal and 1 otherwise, and keeping the last symbol if w has odd length. We can define squish recursively as follows:

$$\text{squish}(w) := \begin{cases} w & \text{if } w = \varepsilon \text{ or } w = 0 \text{ or } w = 1 \\ 0 \cdot \text{squish}(x) & \text{if } w = 00x \text{ or } w = 11x \text{ for some string } x \\ 1 \cdot \text{squish}(x) & \text{if } w = 01x \text{ or } w = 10x \text{ for some string } x \end{cases}$$

For example,

0 1 1 1

$$\text{squish}(\underbrace{00}\underbrace{10}\underbrace{11}\underbrace{10}\underbrace{01}1) = 010111$$

#(1,w) = number of 1s in w

induction

(a) ***Prove*** that $\#(1, \text{squish}(w)) \le \#(1, w)$ for every string w.

(b) ***Prove*** that $\#(1, \text{squish}(w))$ is even if and only if $\#(1, w)$ is even (or equivalently, that $\#(1, \text{squish}(w)) \bmod 2 = \#(1, w) \bmod 2$) for every string w.

As usual, you can assume any result proved in class, in the lecture notes, in labs, in lab solutions, or in homework solutions. In particular, you may use the fact that $\#(1, xy) = \#(1, x) + \#(1, y)$ for all strings x and y.

4. Let L be the set of all strings in $\{0, 1\}^*$ in which every run of 0s is followed immediately by a *shorter* run of 1s. For example, the strings 0001100000111 and 11100100000111 and 11111 are in L, but the strings 00011111 and 000110000 are not.

too long

(a) ***Prove*** that L is *not* a regular language. ← fooling set

(b) Describe a context-free grammar for L.

5. For each of the following languages L over the alphabet $\Sigma = \{0, 1\}$, describe a DFA that accepts L **and** give a regular expression that represents L. You do not need to justify your answers.

(a) Strings that do not contain the subsequence 01110.

(b) Strings that contain at least two even-length runs of 1s.

CS/ECE 374 A ✧ Fall 2025

Midterm 1 Practice 2

September 26, 2025

Name:	Jeff E
NetID:	jeffe

- ***Don't panic!***
- You have 120 minutes to answer five questions. The questions are described in more detail in a separate handout.
- If you brought anything except your writing implements, your **hand-written** double-sided 8½" × 11" cheat sheet, and your university ID, please put it away for the duration of the exam. In particular, please turn off and put away *all* medically unnecessary electronic devices.
- Please clearly print your name and your NetID in the boxes above.
- Please also print your name at the top of every page of the answer booklet, except this cover page. We want to make sure that if a staple falls out, we can reassemble your answer booklet. (It doesn't happen often, but it does happen.)
- Proofs or other justifications are required for full credit if and only if we explicitly ask for them, using the word ***prove*** or ***justify*** in bold italics.
- **Do not write outside the black boxes on each page**. These indicate the area of the page that our scanners will actually scan. If the scanner can't see your work, we can't grade it.
- If you run out of space for an answer, please use the overflow/scratch pages at the back of the answer booklet, but **please clearly indicate where we should look**. If we can't find your work, we can't grade it.
- **Only work that is written into the stapled answer booklet will be graded**. In particular, you are welcome to detach scratch pages from the answer booklet, but any work on those detached pages will not be graded. Please let us know if you detach a page accidentally. We will provide additional scratch paper on request.
- Please return ***all*** paper with your answer booklet: your question sheet, your cheat sheet, and all scratch paper. **Please put all loose paper *inside* your answer booklet.**

Is between
between is and between
or between between and is
or between and and or
or between or and between
or between between and and
or between or and and
or between or and or?

CS/ECE 374 A ✧ Fall 2025 Midterm 1 Practice 2 Problem 1	Name: Jeff E

For each statement below, check "Yes" if the statement is always true and check "No" otherwise, and give a brief (one short sentence) explanation of your answer. Read these statements very carefully—small details matter!

(a) Every infinite language is regular.

☐ Yes ☒ No — $\{0^n1^n \mid n \geq 0\}$ inf not regular.

(b) The language $(0 + 1(01^*0)^*1)^*$ is not context-free.

☐ Yes ☒ No — Every regular language is CF.

(c) Every subset of an irregular language is irregular.

☐ Yes ☒ No — $\emptyset \subseteq \{0^n1^n \mid n \geq 0\}$

(d) The language $\{0^a1^b \mid a - b \text{ is divisible by } 374\}$ is regular.

☒ Yes ☐ No — 374-state DFA state $= (\#0 - \#1) \bmod 374$

(e) If language L is not regular, then L has a finite fooling set.

☒ Yes ☐ No — $\emptyset$ is a fooling set for every language.

(f) If there is a DFA that rejects every string in language L, then L is regular.

☐ Yes ☒ No — DFA rejects everything + $L = \{0^n1^n \mid n \geq 0\}$

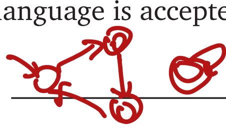
(g) If language L is accepted by an DFA with n states, then its complement $\Sigma^* \setminus L$ is also accepted by a DFA with n states.

☒ Yes ☐ No — $A' = Q \setminus A$

(h) 1^*0^* is a fooling set for the language $\{1^i0^{i+j}1^j \mid i, j \geq 0\}$.

☐ Yes ☒ No — $10^{0^j1^j}$ and $1100^{0^j1^j}$ have no distinguishing suffix

(i) Every regular language is accepted by a DFA with an odd number of accepting states.

☒ Yes ☐ No —  if even, add an inaccessible accepting state.

(j) The context-free grammar $S \to \varepsilon \mid 0S1S \mid 1S0S$ generates all strings in which the number of 0s equals the number of 1s.

☒ Yes ☐ No — HW 1.1

CS/ECE 374 A ✧ Fall 2025 Midterm 1 Practice 2 Problem 2	Name:

For any string w, let cycleleft(w) denote the string obtained by moving the first symbol of w (if any) to the end. (See the question handout for a formal recursive definition.) Let L be an arbitrary regular language over the alphabet $\{0, 1\}$.

input

(a) Prove that $\textsc{CycleLeft}(L) = \{\text{cycleleft}(w) \mid w \in L\}$ is a regular language.

(b) Prove that $\textsc{CycleRight}(L) = \{w \in \Sigma^* \mid \text{cycleleft}(w) \in L\}$ is a regular language.

transformation

(a) Let $M = (Q, s, A, \delta)$ be any DFA for L

We build NFA $M' = (Q', S', A', \delta')$ for $CL(L)$: multiple start state

start:
- guess last input bit ℓ, pass ℓ to M
- when M' reads ℓ later, maybe last bit? if not pass ℓ to M
- when M' reads other bit not ℓ, pass to M.

M' gets 10100
↓
M gets 01010

$$Q' = \{(q, \ell, end?) \mid Q \in q,\ \ell \in \{0,1\},\ end? \in \{T, F\}\}$$
$$= Q \times \{0,1\} \times \{T, F\}$$

$$S' = \{(\delta(s,0), 0, F), (\delta(s,1), 1, F)\}$$

$$A' = \{(q, *, T) \mid q \in A\}$$

$$\delta'((q,0,F), 0) = \{(\delta(q,0), 0, F), (q, 0, T)\}$$
$$\delta'((q,0,F), 1) = \{(\delta(q,1), 0, F)\}$$
$$\delta'((q,1,F), 0) = \{(\delta(q,0), 1, F)\}$$
$$\delta'((q,1,F), 1) = \{(\delta(q,1), 1, F), (q, 1, T)\}$$
$$\delta((q,\ell,T), 0) = \delta((q,\ell,T), 1) = \emptyset$$

See page 7 for part (b)

CS/ECE 374 A ✧ Fall 2025 Midterm 1 Practice 2 Problem 3	Name:

For any string $w \in \{0, 1\}^*$, let squish(w) denote the string obtained by dividing w into pairs of symbols, replacing each pair with 0 if the symbols are equal and 1 otherwise, and keeping the last symbol if w has odd length. (See the question handout for a formal recursive definition.)

LATER.

(a) ***Prove*** that $\#(1, \text{squish}(w)) \le \#(1, w)$ for every string w.

(b) ***Prove*** that $\#(1, \text{squish}(w))$ is even if and only if $\#(1, w)$ is even, for every string w.

induction

(a) Let w be ~~an~~ an arbitrary string

Assume $\#(1, \text{squish}(x)) \le \#(1, x)$ for all x shorter than w.

Cases:

- $|w| \le 1$ $\quad \#(1, \text{squish}(w)) = \#(1, w)$ def. squish

- $w = 00x$ for some string x $\qquad w = aax$

$$\begin{aligned}\#(1, \text{squish}(w)) &= \#(1, \text{squish}(00x)) \\ &= \#(1, 0 \cdot \text{squish}(x)) \\ &= \#(1, \text{squish}(x)) \\ &\le \#(1, x) \quad \text{IH} \\ &= \#(1, 00x) \\ &= \#(1, w)\end{aligned}$$

- $w = 11x$ for some string x

$$\begin{aligned}\#(1, sq(w)) &= \#(1, sq(11x)) \\ &= \#(1, 0 \cdot sq(x)) \\ &= \#(1, sq(x)) \\ &\le \#(1, x) \quad \text{IH} \\ &\le 2 + \#(1, x) \\ &= \#(1, 11x) \\ &= \#(1, w)\end{aligned}$$

see page 6

CS/ECE 374 A ✧ Fall 2025 **Midterm 1 Practice 2 Problem 4**	Name:

Let L be the set of all strings in $\{0, 1\}^*$ in which every run of 0s is followed immediately by a *shorter* run of 1s.

(a) ***Prove*** that L is *not* a regular language.

(b) Describe a context-free grammar for L.

fooling sets

CFG

(a) Think about $\underbrace{L \cap 0^*1^*}_{L'} = \{0^a 1^b \mid a \geq 1, b \geq 1, a > b\}$

Let $F = 0^+$

Let $x \neq y$ be strings in F

Then $x = 0^i$ and $y = 0^j$ where $i \neq j$

WLOG $i < j$ (otherwise swap $x \leftrightarrow y$ and $i \leftrightarrow j$)

Let $z = 1^i$

$xz = 0^i 1^i \notin L$

$yz = 0^j 1^i \in L$ because $i < j$.

So F is infinite fooling set for L ✓

(b) $S \to AB$

$\underbrace{(\text{run1s}+\varepsilon)}_{A}\underbrace{(\text{run0s shorterrun1s})^*}_{B}$

$A \to \varepsilon \mid 1A$ $\quad 1^*$

$B \to \varepsilon \mid CB$

$C \to DE$

$D \to 0 \mid 0D$ $\quad 0^+$

$E \to 01 \mid 0E1$ $\quad \{0^n 1^n \mid n \geq 1\}$

run0s shorterrun1s

= longerrun0s run1s

$= \underbrace{\text{run0s}}_{D}\underbrace{(0^n 1^n)}_{E}$

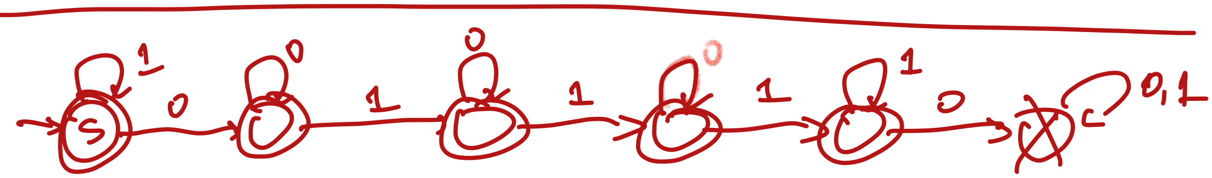
CS/ECE 374 A ✧ Fall 2025 Midterm 1 Practice 2 Problem 5	Name:

For each of the following languages L over the alphabet $\Sigma = \{0, 1\}$, describe a DFA that accepts L **and** give a regular expression that represents L. You do not need to justify your answers.

(a) Strings that do not contain the subsequence `01110`.

(b) Strings that contain at least two even-length runs of `1`s.

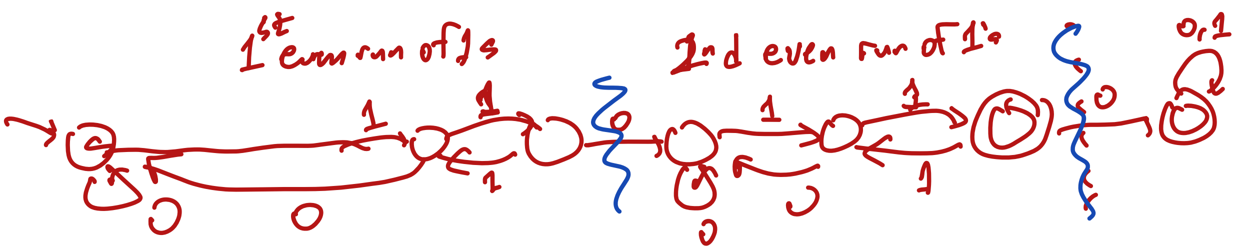
(a)



- No 0s
- ...0...≤2 1s...
- ..0..3 1s/no 0s

$$\boxed{1^* + 1^*0(0^*10^*10^*) + 1^*0\,0^*10^*10^*1\,1^*}$$

(b)



$$\underbrace{0^*(1(11)^*00^*)^*11(11)^*}_{\text{ends with even run of 1's}}\;0\;\underbrace{0^*(1(11)^*00^*)\,11(11)^*}\;\cdot(\varepsilon+0(0+1)^*)$$

ends with
even run of 1's

#3a

- $w = 01x$ or $10x$

$$\begin{aligned}
\#(1, sq(w)) &= \#(1, sq(abx)) \\
&= \#(1, 1 \cdot sq(x)) \\
&= 1 + \#(1, sq(x)) \\
&\le 1 + \#(1, x) \quad \text{IH} \\
&= \#(1, abx) \\
&= \#(1, w)
\end{aligned}$$

In all cases $\#1(sq(w)) \le \#1(w)$ ✓

3b Prove $\#(1, sq(w)) \bmod 2 = \#(1, w) \bmod 2$

Let w be any string

Assume $\#(1, sq(x)) \bmod 2 = \#(1, x) \bmod 2$ for all x shorter than w

Cases:

- $|w| \le 1 \longrightarrow \#(1, sq(w)) = \#(1, w)$ ✓
- $w = aax$ for some symbol a and string x

$$\begin{aligned}
\#(1, sq(w)) \bmod 2 &= \#(1, sq(aax)) \bmod 2 \\
&= \#(1, 0 \cdot sq(x)) \bmod 2 \\
&= \#(1, sq(x)) \bmod 2 \\
&= \#(1, x) \bmod 2 \quad \leftarrow \text{IH} \\
&= \#(1, aax) \bmod 2 \\
&= \#(1, w) \bmod 2
\end{aligned}$$

- $w = abx$ where $a \ne b$

$$\begin{aligned}
\#(1, sq(w)) \bmod 2 &= \#(1, sq(ab\cancel{x})) \bmod 2 \\
&= \#(1, 1 \cdot sq(\cancel{x})) \bmod 2 \\
&= 1 + \#(1, sq(x)) \bmod 2 \\
&= 1 + \#(1, x) \bmod 2 = \cancel{2}\#(1, abx) \bmod 2 \\
&= \#(w) \bmod 2 \checkmark
\end{aligned}$$

2b

(scratch paper)

Let $M=(Q,s,A,\delta)$ be DFA for L

We build NFA $M'=(Q',s',A',\delta')$ for $CR(L)$ as follows:

M' reads 11000

M reads 10001

$$Q' = Q \times \{\text{start}, \text{hold}0, \text{hold}1, \text{end}\}$$

$$s' = (s, \text{start})$$

$$A' = A \times \{\text{done}\} = \{(q, \cancel{\text{done}}\ \text{end}) \mid q \in A\}$$

$$\delta'((q,\text{start}), a) = \{(q, \text{hold}\,a)\}$$

$$\delta'((q,\text{start}), \varepsilon) = \{(q, \text{end})\}$$

$$\delta'((q,\text{hold}0), a) = \{(\delta(q,a), \text{hold}0)\}$$

$$\delta'((q,\text{hold}1), a) = \{(\delta(q,a), \text{hold}1)\}$$

$$\delta'((q,\text{hold}0), \varepsilon) = \{(\delta(q,0), \text{end})\}$$

$$\delta'((q,\text{hold}1), \varepsilon) = \{(\delta(q,1), \text{end})\}$$

all missing transitions go to $\emptyset$

(scratch paper)