CS/ECE 374 A ✧ Fall 2025

Midterm 1 Practice 1

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- ***Don't panic!***
- You have 120 minutes to answer five questions. The questions are described in more detail in a separate handout.
- If you brought anything except your writing implements, your **hand-written** double-sided 8½" × 11" cheat sheet, and your university ID, please put it away for the duration of the exam. In particular, please turn off and put away *all* medically unnecessary electronic devices.
- Please clearly print your name and your NetID in the boxes above.
- Please also print your name at the top of every page of the answer booklet, except this cover page. We want to make sure that if a staple falls out, we can reassemble your answer booklet. (It doesn't happen often, but it does happen.)
- Proofs or other justifications are required for full credit if and only if we explicitly ask for them, using the word ***prove*** or ***justify*** in bold italics.
- **Do not write outside the black boxes on each page**. These indicate the area of the page that our scanners will actually scan. If the scanner can't see your work, we can't grade it.
- If you run out of space for an answer, please use the overflow/scratch pages at the back of the answer booklet, but **please clearly indicate where we should look**. If we can't find your work, we can't grade it.
- **Only work that is written into the stapled answer booklet will be graded**. In particular, you are welcome to detach scratch pages from the answer booklet, but any work on those detached pages will not be graded. Please let us know if you detach a page accidentally. We will provide additional scratch paper on request.
- Please return ***all*** paper with your answer booklet: your question sheet, your cheat sheet, and all scratch paper. **Please put all loose paper *inside* your answer booklet.**

Is between
between is and between
or between between and is
or between and and or
or between or and between
or between between and and
or between or and and
or between or and or?

CS/ECE 374 A ✧ Fall 2025 Midterm 1 Practice 1 Problem 1	Name: Emily Fox

Consider the function compress0s defined in the question handout. Let L be an arbitrary regular language.

(a) ***Prove*** that $\{w \in \Sigma^* \mid \text{compress0s}(w) \in L\}$ is regular.

(b) ***Prove*** that $\{\text{compress0s}(w) \mid w \in L\}$ is regular.

(a) Let $M = (Q, s, A, \delta)$ be a DFA accepting L.

Build NFA $M' = (Q', S', A', \delta')$.

$Q' = Q \times \{no, yes\}$

$S' = \{(s, no)\}$

$A' = \{(q, no), (q, yes)\} \ \forall q \in A$

$\delta'((q, no), 0) = \{(\delta(q, 0), yes)\}$

$\delta'((q, no), 1) = \{(\delta(q, 1), no)\}$

$\delta'((q, yes), 0) = \{(q, no)\}$

$\delta'((q, yes), 1) = \{(\delta(q, 1), no\}$

(b) Let $M = (Q, s, A, \delta)$ be a DFA accepting L.

Build an NFA $M' = (Q', S', A', \delta')$.

$Q' = Q \times \{can, cannot\}$

M can eat a 0 + must if next input char is 0

$S' = \{(s, cannot)\}$

$A' = (A \times \{can, cannot\}) \cup \{(q, can) \mid \delta(q, 0) \in A\}$

$\delta'((q, cannot), 0\} = \{(\delta(q, 0), can)$

$\delta'((q, cannot), 1\} = \{\delta(q, 1), cannot)\}$

$\delta'((q, can), 0\} = \{(\delta(\delta(q, 0), 0), can)\}$

$\delta'((q, can), 1\} = \{(\delta(\delta(q, 0), 1), cannot), (\delta(q, 1), cannot)\}$

Alternative for (b) on page 7. (We only grade first sol.)

CS/ECE 374 A ✧ Fall 2025 Midterm 1 Practice 1 Problem 2	Name: Emily Fox

Let L be the language of all strings over $\{0, 1\}$ that contain at least 374 consecutive 1s.

(a) Give a regular expression that matches L. *[Hint: Use the notation R^k to denote the concatenation of k copies of R.]*

(b) Describe a DFA whose language is L. *[Hint: Do not try to **draw** your DFA!]*

(c) ***Prove*** that any DFA whose language is L must have at least 375 states, using a fooling set argument.

(a) $(0+1)^* 1^{374} (0+1)^*$

(b) $M = (Q, s, A, \delta)$ s.t. $Q = \{0, 1, \dots, 374\}$

$s = 0$

$A = \{374\}$

$\delta(q, 1) = q+1 \quad (q \neq 374)$

$\delta(374, 0) = 374, \ \delta(374, 1) = 374$

$\delta(q, 0) = 0 \quad (q \neq 374)$

(c) Let $F = \{1^n \mid 0 \leq n \leq 374\}$.

Let $x, y \in F$ be distinct.

$x = 1^i$ and $y = 1^j$ with $i \neq j$. Assume $i < j$.

Let $z = 1^{374-j}$

$xz = 1^{i+374-j} \notin L$, because $i < j \Rightarrow i + 374 - j < 374$,

$yz = 1^{374} \in L$

We distinguished arbitrary $x \neq y$ in F so F is a fooling set.

$|F| \geq 375$, so any DFA needs ≥ 375 states.

CS/ECE 374 A ✧ Fall 2025 **Midterm 1 Practice 1 Problem 3**	Name: Emily Fox

Consider the recursive function Bond defined in the question handout.

(a) ***Prove*** that $|\text{Bond}(w)| \geq |w|$ for all strings w.

(b) ***Prove*** that $\text{Bond}(x \bullet y) = \text{Bond}(x) \bullet \text{Bond}(y)$ for all strings x and y.

(a) Let w be an arbitrary string.
Assume for strings x s.t. $|x| < |w|$, $|\text{Bond}(x)| \geq |x|$.
There are three cases.
Suppose $w = \varepsilon$

$|\text{Bond}(w)| = |\text{Bond}(\varepsilon)|$
$= |\varepsilon|$
$= |w|$

Suppose $w = 0x$.

$|\text{Bond}(w)| = |\text{Bond}(0x)|$
$= |00 \cdot \text{Bond}(x)|$
$= 2 + |\text{Bond}(x)|$
$\geq 2 + |x|$
$> 1 + |x|$
$= |0x|$
$= |w|$

continued on scratch 6

(b) Let x and y be arbitrary strings.
Assume for all strings z s.t. $|z| < |x|$,
$\text{Bond}(z \bullet y) = \text{Bond}(z) \bullet \text{Bond}(y)$
Suppose $x = \varepsilon$.

$\text{Bond}(x \bullet y) = \text{Bond}(\varepsilon \bullet y)$
$= \varepsilon \bullet \text{Bond}(y)$
$= \text{Bond}(\varepsilon) \bullet \text{Bond}(y)$
$= \text{Bond}(x) \bullet \text{Bond}(y)$

Suppose $x = 0z$

$\text{Bond}(x \bullet y) = \text{Bond}((0z) \bullet y)$
$= 00 \cdot \text{Bond}(z \bullet y)$
$= 00 \cdot \text{Bond}(z) \bullet \text{Bond}(y)$
$= \text{Bond}(0z) \bullet \text{Bond}(y)$
$= \text{Bond}(x) \bullet \text{Bond}(y)$

 continued on 6

CS/ECE 374 A ✧ Fall 2025 Midterm 1 Practice 1 Problem 4	Name: Emily Fox

Let L be the language $\{0^a 1^b 0^c \mid a = b \text{ or } a = c \text{ or } b = c\}$

a 1. ***Prove*** that L is *not* a regular language.

2. Describe a context-free grammar for L. You do not need to justify your answer.

(a) Let $F = \{1^n \mid n \geq 1\}$.

Let $x, y \in F$ be distinct.

$x = 1^i$ and $y = 1^j$ for $i \neq j$, $i \geq 1$, $j \geq 1$.

Let $z = 0^i$.

$xz = 1^i 0^i \in L$, but $yz = 1^j 0^i \notin$, because $i \neq j$.

x and y were arbitrary members of F, so F is a fooling set.

F is infinite, so L is not regular.

(b) $S \rightarrow A \mid BE \mid EC$

$A \rightarrow 0A0 \mid D$

$B \rightarrow \varepsilon \mid 0B1$

$C \rightarrow \varepsilon \mid 1C0$

$D \rightarrow \varepsilon \mid 1D$

$E \rightarrow \varepsilon \mid 0E$

L

$0^a 1^b 0^a$

$0^a 1^a 0^c$

$0^a 1^b 0^b$

1^b

0^a

CS/ECE 374 A ✧ Fall 2025 Midterm 1 Practice 1 Problem 5	Name: Emily Fox

For each statement below, check "Yes" if the statement is ***always*** true and check "No" otherwise, and write a ***brief*** (one short sentence) explanation of your answer. Read these statements very carefully—small details matter!

For any string $w \in (0+1)^*$, let w^C denote the *bitwise complement* of w, obtained by flipping every 0 in w to a 1, and vice versa. For example, $\varepsilon^C = \varepsilon$ and $000110^C = 111001$.

(a) If $2+2=5$, then zero is odd.

Yes No

hypothesis is false

(b) $\{0^n1 \mid n > 0\}$ is the only infinite fooling set for the language $\{0^n10^n \mid n > 0\}$.

Yes No

$\{0^n10 \mid n > 5\}$

(c) $\{0^n10^n \mid n > 0\}$ is a context-free language.

Yes No

$S \to 010,\ 0S0$

(d) The context-free grammar $S \to 00S \mid S11 \mid 01$ generates the language $\{0^n1^n \mid n \geq 0\}$.

Yes No

ε not generated

(e) Every regular language is recognized by a DFA with exactly one accepting state.

Yes No

$L = \{w \mid |w| \not\equiv 0 \bmod 3\}$

(f) Any language that can be decided by an NFA with ε-transitions can also be decided by an NFA without ε-transitions.

Yes No

add ε-reaches to transitions

(g) If L is a regular language over the alphabet $\{0, 1\}$, then $\{xy^C \mid x, y \in L\}$ is also regular.

y^C regular

Yes No

$L \cdot L^C$ is regular

(h) If L is a regular language over the alphabet $\{0, 1\}$, then $\{ww^C \mid w \in L\}$ is also regular.

Yes No

$L = 0^*$ leads to 0^n1^n

(i) The regular expression $(00 + 11)^*$ represents the language of all strings over $\{0, 1\}$ of even length.

Yes No

01 is not represented

(j) Let L_1, L_2 be two regular languages. The language $(L_1 + L_2)^*$ is also regular.

Yes No

def. of regular

(scratch paper)

3(a) cont.

Suppose $w = 1x1$

$|Bond(w)| = |Bond(1x)|$

$= |1 \cdot Bond(x)|$

$= 1 + |Bond(x)|$

$\geq 1 + |x|$

$= |1x|$

$= |w|$

In all cases,

$|Bond(w)| \geq |w|$

3(b) cont.

Suppose $x = 1z$

$Bond(x \cdot y) = Bond(1zy)$

$= 1 \cdot Bond(z \cdot y)$

$= 1 \cdot Bond(z) \cdot Bond(y)$

$= Bond(1z) \cdot Bond(y)$

$= Bond(x) \cdot Bond(y)$

In all cases,

$Bond(x \cdot y) = Bond(x) \cdot Bond(y)$

1(6) alternatie (we only grade first solution given)

Let $M = (Q, s, A, \delta)$ accept L.

We build NFA with ϵ-transitions $M' = (Q', S', A', \delta')$

$Q' = Q \times \{hungry, full\}$

M' won't see the next 0

M will see next 0

$S = \{(s, full)\}$

$A' = A \times \{hungry, full\}$

$\delta'((q, hungry), 0) = \emptyset$

$\delta'((q, full), 0) = \{\delta(q, 0), hungry)\}$

$\delta'((q, hungry), 1) = \{\delta(q, 1), full)\}$

$\delta'((q, full), 1) = \{\delta(q, 1, full)\}$

$\delta'((q, hungry), \epsilon) = \{(\delta(q, 0), full)\}$

$\delta'((q, full), \epsilon) = \emptyset$

(scratch paper)