

GPS 1 due 9pm today

HW 1 due 9pm tomorrow

homework policies on website

cite all sources or say none

transcripts with CLMs

Standard rubrics on website

All strings $w \in \{0,1\}^n$ s.t. $\#(1, w) \equiv 0 \pmod{3}$?

Nums Div By 3 ($w[1..n]$)

rem $\leftarrow 0$

for $i \leftarrow 1$ to n

if $w[i] = 1$

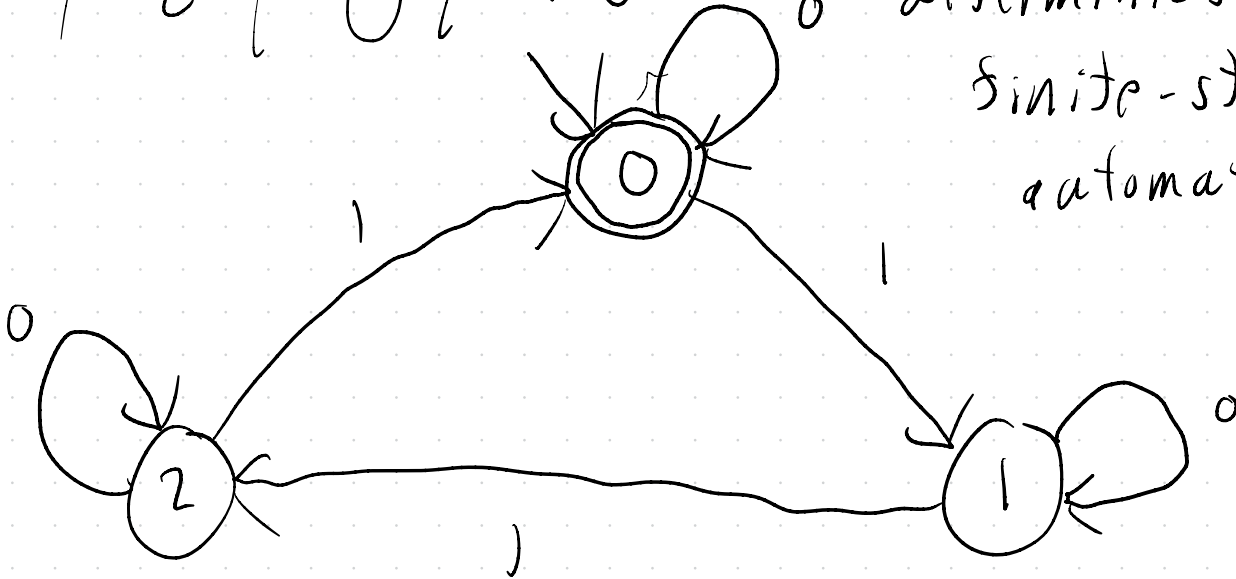
rem $\leftarrow (\text{rem} + 1) \pmod{3}$

return (rem = 0)

	0	1	return trap?
0	0	1	yes
1	1	2	no
2	2	0	no

finite-state machine

deterministic finite-state automata (DFA)



A DFA consists of five components:

- an arbitrary finite set Σ , the input alphabet
- an arbitrary finite set Q of states
- an arbitrary transition function:

$$\delta: Q \times \Sigma \rightarrow Q$$

- a start state $s \in Q$

- a subset $A \subseteq Q$ of accept states

$$\text{ex: } \Sigma = \{0, 1\}$$

$$Q = \{0, 1, 2\}$$

$$\delta(q, a) = \begin{cases} q & \text{if } a = 0 \\ (q+1) \bmod 3 & \text{if } a = 1 \end{cases}$$

$$s = 0$$

$$A = \{0\}$$

extended transition function

$$\delta^* : Q \times \Sigma^* \rightarrow Q$$

$$\delta^*(q, w) := \begin{cases} q & \text{if } w = \epsilon \\ \delta^*(\delta(q, a), x) & \text{if } w = ax \end{cases}$$

DFA accepts w iff $\delta^*(s, w) \in A$

rejects w otherwise

$$\delta^*(q, 01010110101) = \delta^*(q, 1010110101)$$

$$= \delta^*(1, 010110101)$$

$$= \delta^*(1, 10110101)$$

⋮

$$= \delta^*(2, 01)$$

$$= \delta^*(2, 1)$$

$$= \delta^*(0, \epsilon)$$

$$= 0$$

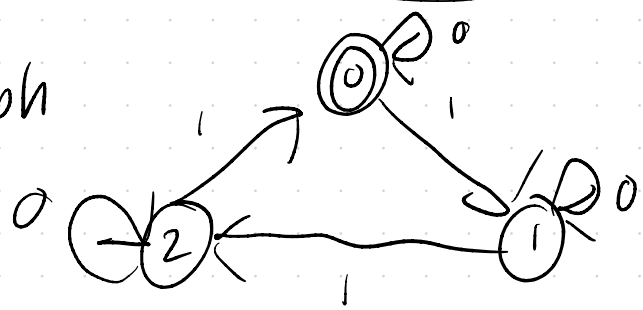
$$\in A$$

accept!

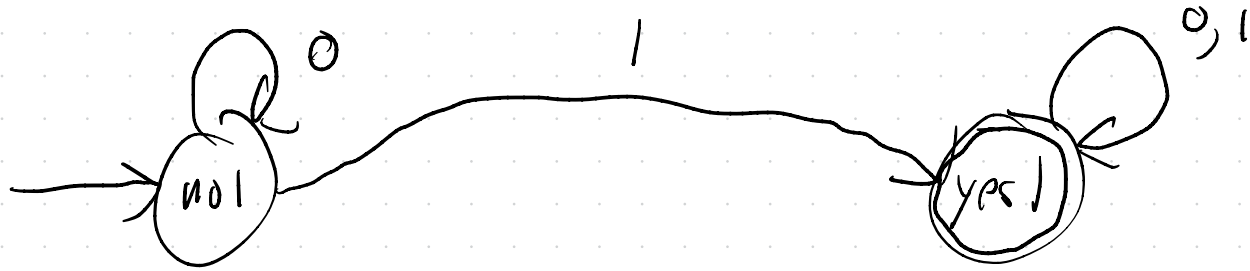
q	$\delta(q, 0)$	$\delta(q, 1)$	$q \in A?$
0	0	1	True
1	1	2	False
2	2	0	False

start at 0

or the graph



Binary strings contain a 1...



All strings with 11 as a substring:

CONTAINS11($w[1..n]$):

$found \leftarrow \text{FALSE}$

for $i \leftarrow 1$ to n

 if $i = 1$

$last2 \leftarrow w[1]$

 else

$last2 \leftarrow w[i-1] \cdot w[i]$

 if $last2 = 11$

$found \leftarrow \text{TRUE}$

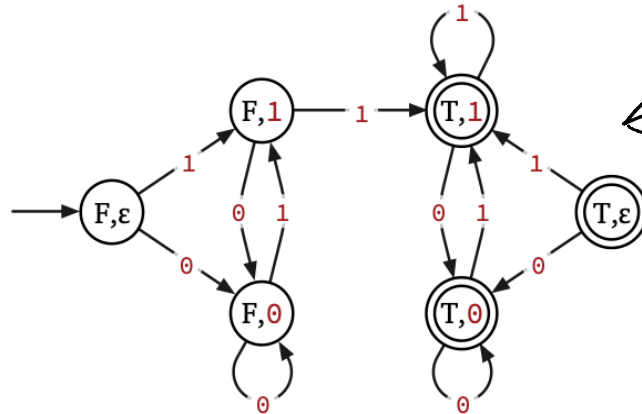
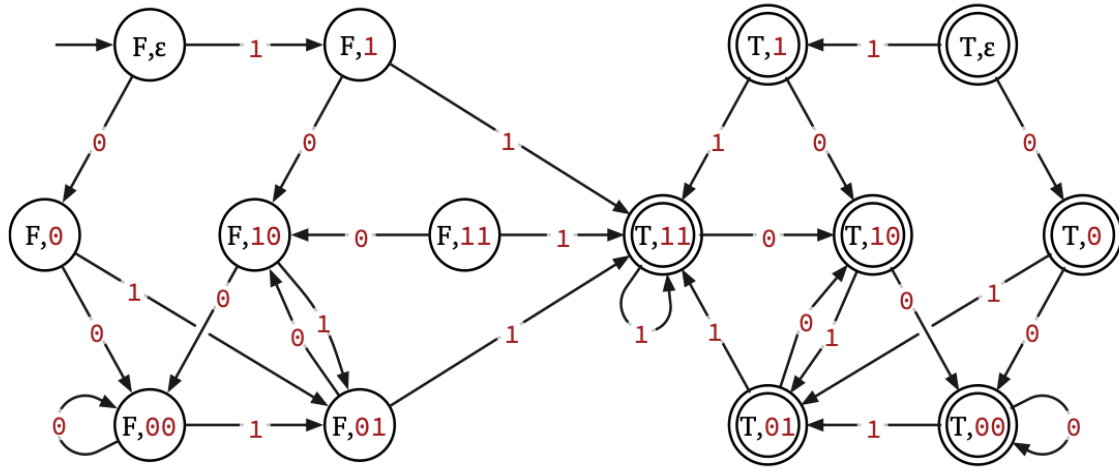
return $found$

two variables : found a Boolean
 last 2 a string of length
 ≤ 2

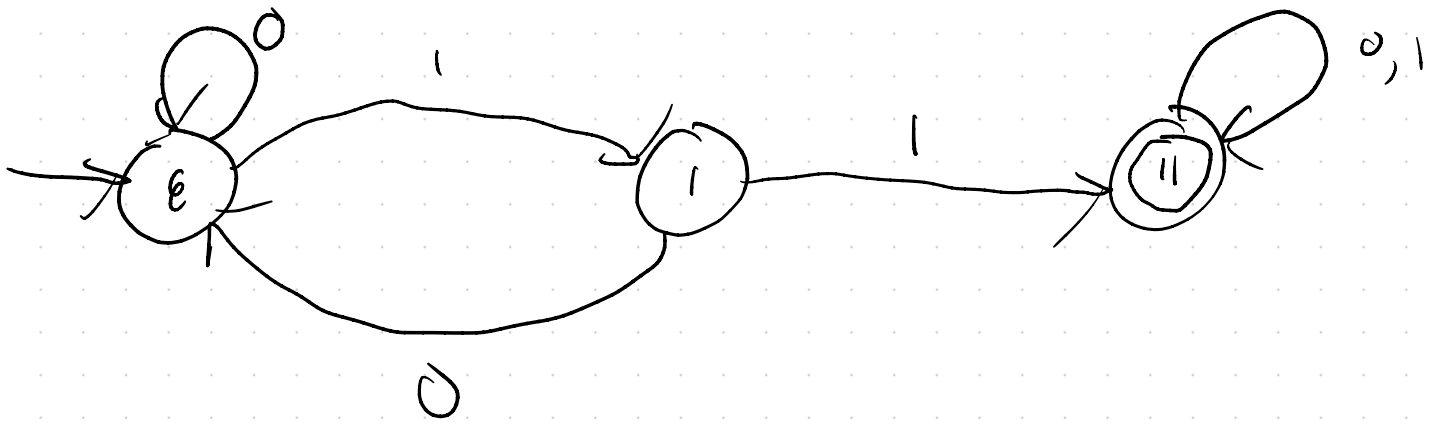
so $2 \cdot 7 = 14$

q	$\delta[q, 0]$	$\delta[q, 1]$	q	$\delta[q, 0]$	$\delta[q, 1]$
(FALSE, ϵ)	(FALSE, 0)	(FALSE, 1)	(TRUE, ϵ)	(TRUE, 0)	(TRUE, 1)
(FALSE, 0)	(FALSE, 00)	(FALSE, 01)	(TRUE, 0)	(TRUE, 00)	(TRUE, 01)
(FALSE, 1)	(FALSE, 10)	(TRUE, 11)	(TRUE, 1)	(TRUE, 10)	(TRUE, 11)
(FALSE, 00)	(FALSE, 00)	(FALSE, 01)	(TRUE, 00)	(TRUE, 00)	(TRUE, 01)
(FALSE, 01)	(FALSE, 10)	(TRUE, 11)	(TRUE, 01)	(TRUE, 10)	(TRUE, 11)
(FALSE, 10)	(FALSE, 00)	(FALSE, 01)	(TRUE, 10)	(TRUE, 00)	(TRUE, 01)
(FALSE, 11)	(FALSE, 10)	(TRUE, 11)	(TRUE, 11)	(TRUE, 10)	(TRUE, 11)

start at
 (False, ϵ)
 accept
 (True, \bullet)



← only remember
one last
character



All numbers w that are a multiple of 5.

$$\begin{array}{r}
 0110 \\
 \times 2 \\
 \hline
 \end{array}
 \begin{array}{l}
 \downarrow \\
 + \uparrow
 \end{array}$$

MULTIPLEOF5(w[1..n]):

```
rem ← 0
for i ← 1 to n
    rem ← (2 · rem + w[i]) mod 5
if rem = 0
    return TRUE
else
    return FALSE
```

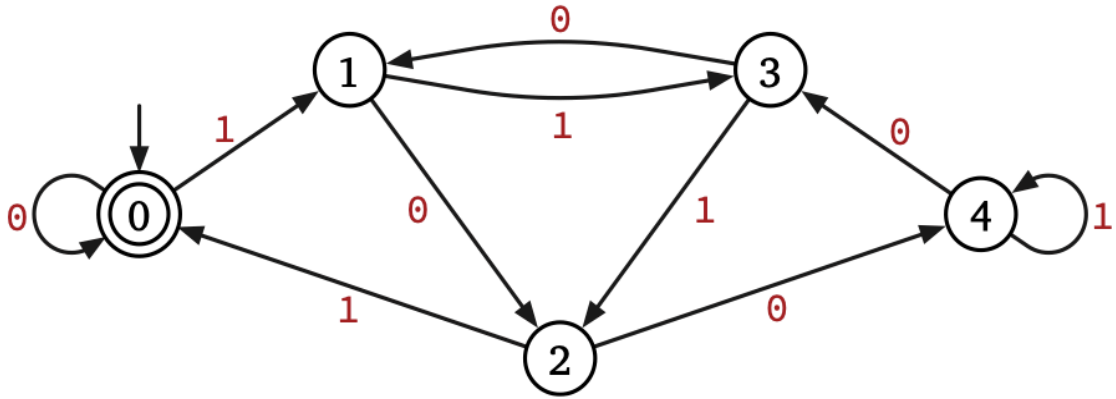
$$Q = \{0, 1, 2, 3, 4\}$$

$$\delta(q, a) =$$

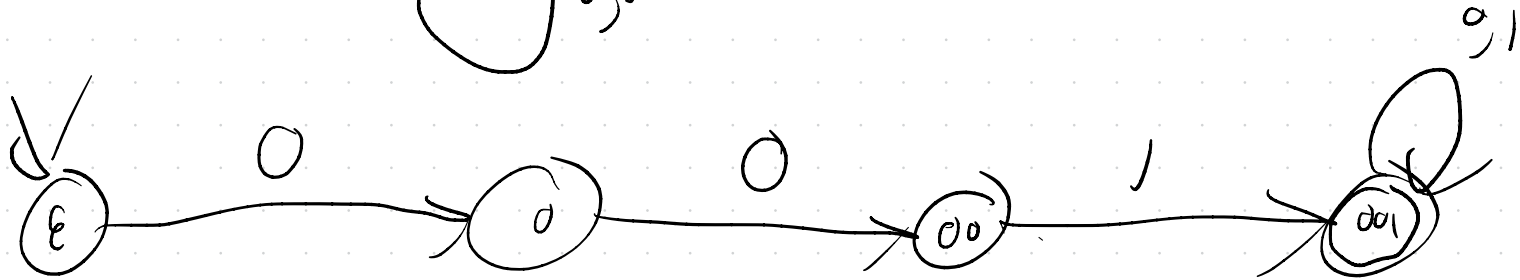
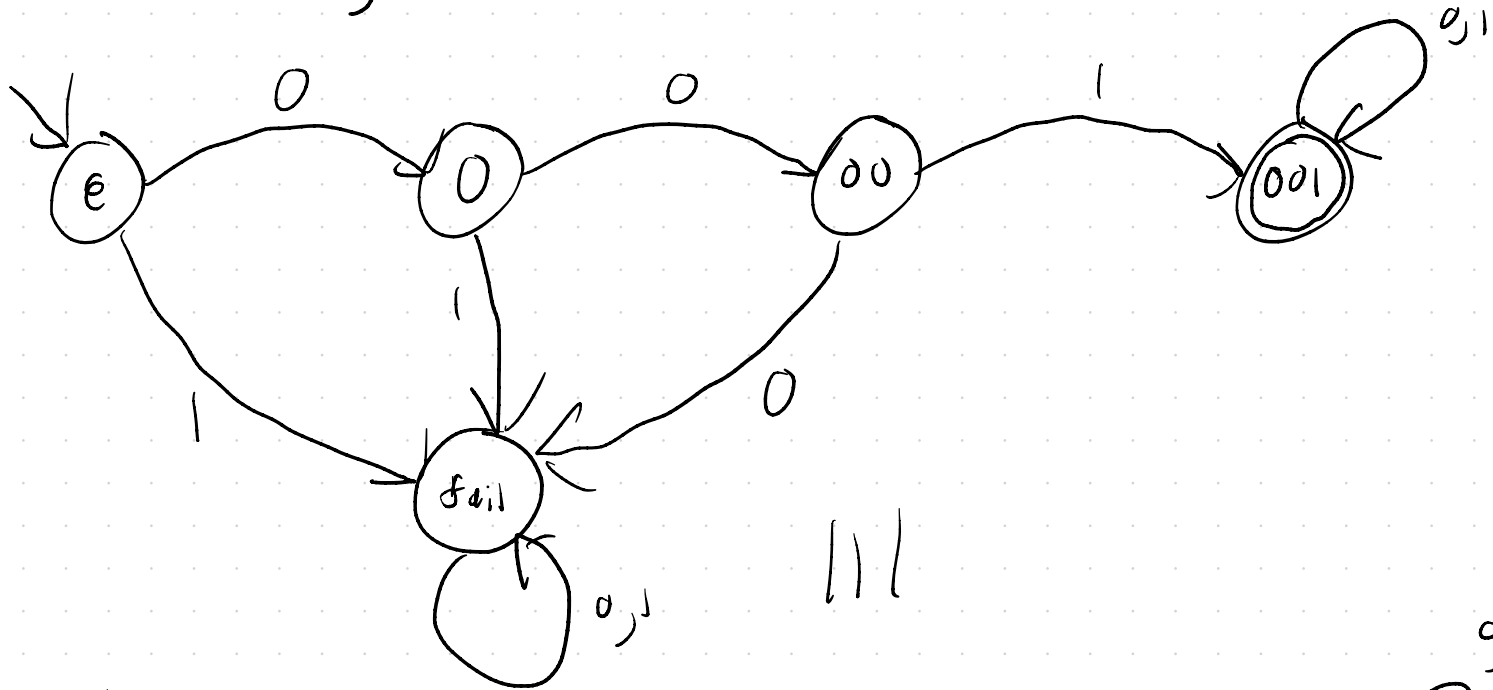
$$(2q + a) \text{ mod } 5$$

start at 0

q	$\delta[q, 0]$	$\delta[q, 1]$	$A[q]$
0	0	1	TRUE
1	2	3	FALSE
2	4	0	FALSE
3	1	2	FALSE
4	3	4	FALSE



All strings beginning with 001:



implicit fail if a transition does not exist. Write if you do this!

