Write your answers in the separate answer booklet.
You have 120 minutes (after you get the answer booklet) to answer five questions.
Please return this question sheet and your cheat sheet with your answers.

1. For each statement below, check "Yes" if the statement is ***always*** true and check "No" otherwise, and give a brief (one short sentence) explanation of your answer. Read these statements very carefully—small details matter!

   For any string $w \in \{0, 1\}^*$, let w^C denote the bitwise complement of w; for example, $(00110)^C = 11001$.

   (a) For every finite language L, the language L^* is regular.

   (b) For every finite language L, the language L^* is context-free.

   (c) For every language L, if L^* is regular, then L is regular.

   (d) For every regular language L over $\{0, 1\}$, the language $\{w^C \mid w \in L\}$ is also regular.

   (e) If L has a fooling set of size 374, then every NFA for L requires at least 374 states.

   (f) $(01)^*$ is a fooling set for the language $\{w \in \{0, 1\}^* \mid \#(0, w) = \#(1, w)\}$.

   (g) If F is a fooling set for some regular language L, then F is a regular language.

   (h) The language $\left\{0^a 0^b \mid a = b\right\}$ is regular.

   (i) If language L is accepted by an NFA with n states, then its complement $\Sigma^* \setminus L$ is also accepted by an NFA with n states.

   (j) For every language L with at least two strings, L^* contains infinite-length strings.

2. For any non-empty string w with length at least 2, let $\mathsf{Delete2nd}(w)$ denote the string obtained by deleting the second symbol in w. For example:

   $\mathsf{Delete2nd}(100) = 10$ $\quad$ $\mathsf{Delete2nd}(000111) = 00111$ $\quad$ $\mathsf{Delete2nd}(1010101) = 110101$

   Let L be an arbitrary regular language over the alphabet $\Sigma = \{0, 1\}$. ***Prove*** that the following languages are also regular.

   (a) $\textsc{Insert2nd}(L) = \left\{w \in \Sigma^* \;\middle|\; |w| \geq 2 \text{ and } \mathsf{Delete2nd}(w) \in L\right\}$

   (b) $\textsc{Delete2nd}(L) = \left\{\mathsf{Delete2nd}(w) \;\middle|\; |w| \geq 2 \text{ and } w \in L\right\}$

Questions 3, 4, and 5 are on the back of this page.

3. For each of the following languages over the alphabet {I, L, N}, describe both a regular expression that matches the language and a DFA that accepts the language. You do not need to prove that your answers are correct.

   (a) All strings in $\{\mathtt{I},\mathtt{L},\mathtt{N}\}^*$ where every run of Ls has length 2.

   This language contains the strings ILLINI and ILLILLI and INILLILLININI and the empty string, but it does not contain the strings ILINNI or NILLLLL.

   (b) All strings that are covered by ILL and INI substrings. A string w is in this language if and only if every character in w is contained in a substring of w that is equal to either ILL or INI.

   This language contains the strings ILLINI and INILLINIININI and the empty string, but it does not contain the strings ILLILLI or ILINNI or NILLLL.

4. Recall that $\#(a, w)$ denotes the number of occurrences of symbol a in string w.

   Consider the following recursively defined function on strings over $\{\mathtt{0},\mathtt{1}\}$:

$$\mathsf{grow}(w) = \begin{cases} \varepsilon & \text{if } w = \varepsilon \\ \mathtt{1} \cdot \mathsf{grow}(x) & \text{if } w = \mathtt{0}x \text{ for some string } x \\ \mathtt{01} \bullet \mathsf{grow}(x) & \text{if } w = \mathtt{1}x \text{ for some string } x \end{cases}$$

   For example, $\mathsf{grow}(\mathtt{10001}) = \mathtt{0111101}$.

   Prove that the following inequality holds for all strings $w \in \{\mathtt{0},\mathtt{1}\}^*$:

$$\#(\mathtt{0}, \mathsf{grow}(\mathsf{grow}(w))) \geq \#(\mathtt{0}, \mathsf{grow}(w)).$$

   As usual, you may assume any result proved in class, in the lecture notes, in labs, in lab solutions, or in homework solutions. In particular, you may use the fact that $\#(a, xy) = \#(a, x) + \#(a, y)$ for every symbol a and all strings x and y.

5. Let S be the set of all strings in $(\mathtt{ILL} + \mathtt{INI})^*$ in which the substrings ILL and INI appear the same number of times. For example, S contains the strings ILLINI and INIINIILLILL and the empty string, but S does not contain the strings INIILLILL or INILL or NIL.

   (a) ***Prove*** that S is not a regular language.

   (b) Describe a context-free grammar for S.

   [Hint: Please be careful to write your letters I and L differently from each other and from the vertical bars in your grammar.]