You have 120 minutes to answer five questions.

Write your answers in the separate answer booklet.

Please return this question sheet and your cheat sheet with your answers.

1. For each statement below, check "Yes" if the statement is always true and check "No" otherwise, and give a brief (one short sentence) explanation of your answer. Read these statements very carefully—small details matter!

   (a) Every integer in the empty set is prime.

   (b) The language $\{0^m 1^n \mid m+n \le 374\}$ is regular.

   (c) The language $\{0^m 1^n \mid m-n \le 374\}$ is regular.

   (d) For all languages L, the language L^* is regular.

   (e) For all languages L, the language L^* is infinite.

   (f) For all languages $L \subseteq \Sigma^*$, if L can be represented by a regular expression, then $\Sigma^* \setminus L$ is recognized by a DFA.

   (g) For all languages L and L', if $L \cap L' = \varnothing$ and L' is not regular, then L is regular.

   (h) Every regular language is recognized by a DFA with at least 374 accepting states.

   (i) Every regular language is recognized by an NFA with at most 374 accepting states.

   (j) Every context-free language has an infinite fooling set.

2. The *parity* of a bit-string w is 0 if w has an even number of 1s, and 1 if w has an odd number of 1s. For example:

$$parity(\varepsilon) = 0 \qquad parity(0010100) = 0 \qquad parity(00101110100) = 1$$

   (a) Give a *self-contained*, formal, recursive definition of the *parity* function. (In particular, do ***not*** refer to # or other functions defined in class.)

   (b) ***Prove*** that for every regular language L, the language $\textsc{OddParity}(L) := \{w \in L \mid parity(w) = 1\}$ is also regular.

   (c) ***Prove*** that for every regular language L, the language $\textsc{AddParity}(L) := \{parity(w) \cdot w \mid w \in L\}$ is also regular.

3. For each of the following languages over the alphabet $\Sigma = \{0, 1\}$, either ***prove*** that the language is regular or ***prove*** that the language is not regular. Both of these languages contain the string 00110100000110100.

   (a) $\{0^n w 0^n \mid w \in \Sigma^+ \text{ and } n > 0\}$

   (b) $\{w 0^n w \mid w \in \Sigma^+ \text{ and } n > 0\}$

   [Hint: Exactly one of these two languages is regular.]

4. For any string $w \in \{0, 1\}^*$, let take2skip2(w) denote the subsequence of w containing symbols at positions $1, 2, 5, 6, 9, 10, \ldots 4i+1, 4i+2, \ldots$. In other words, take2skip2(w) takes the first two symbols of w, skips the next two, takes the next two, skips the next two, and so on.For example:

   $$\text{take2skip2}(\underline{1}) = 1$$
   $$\text{take2skip2}(\underline{01}0) = 01$$
   $$\text{take2skip2}(\underline{01}00\underline{11}11\underline{00}01\underline{1}) = 0111001$$

   Let L be an arbitrary regular language over $\{0, 1\}$.

   (a) ***Prove*** that the language $\{w \in \Sigma^* \mid \text{take2skip2}(w) \in L\}$ is regular.

   (b) ***Prove*** that the language $\{\text{take2skip2}(w) \mid w \in L\}$ is regular.

5. For each of the following languages L over the alphabet $\{0, 1\}$, describe a DFA that accepts L **and** give a regular expression that represents L. You do ***not*** need to prove that your answers are correct.

   (a) All strings in which every run of 1s has even length and every run of 0s has odd length. (Recall that a *run* is a maximal substring in which all symbols are equal.)

   (b) All strings in 0^*10^* whose length is a multiple of 3.