ꙮ **Midterm 1 Practice 1** ꙮ

September 25, 2025

Name:	
NetID:	

- *Don't panic!*

- You have 120 minutes to answer five questions. The questions are described in more detail in a separate handout.

- If you brought anything except your writing implements, your **hand-written** double-sided 8½" × 11" cheat sheet, and your university ID, please put it away for the duration of the exam. In particular, please turn off and put away *all* medically unnecessary electronic devices.

- Please clearly print your name and your NetID in the boxes above.

- Please also print your name at the top of every page of the answer booklet, except this cover page. We want to make sure that if a staple falls out, we can reassemble your answer booklet. (It doesn't happen often, but it does happen.)

- Proofs or other justifications are required for full credit if and only if we explicitly ask for them, using the word *prove* or *justify* in bold italics.

- **Do not write outside the black boxes on each page**. These indicate the area of the page that our scanners will actually scan. If the scanner can't see your work, we can't grade it.

- If you run out of space for an answer, please use the overflow/scratch pages at the back of the answer booklet, but **please clearly indicate where we should look**. If we can't find your work, we can't grade it.

- **Only work that is written into the stapled answer booklet will be graded**. In particular, you are welcome to detach scratch pages from the answer booklet, but any work on those detached pages will not be graded. Please let us know if you detach a page accidentally. We will provide additional scratch paper on request.

- Please return *all* paper with your answer booklet: your question sheet, your cheat sheet, and all scratch paper. **Please put all loose paper *inside* your answer booklet.**

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Consider the function compress0s defined in the question handout. Let L be an arbitrary regular language.

(a) **Prove** that $\{w \in \Sigma^* \mid \text{compress0s}(w) \in L\}$ is regular.

(b) **Prove** that $\{\text{compress0s}(w) \mid w \in L\}$ is regular.

Let L be the language of all strings over $\{0, 1\}$ that contain at least 374 consecutive 1s.

(a) Give a regular expression that matches L. *[Hint: Use the notation R^k to denote the concatenation of k copies of R.]*

(b) Describe a DFA whose language is L. *[Hint: Do not try to **draw** your DFA!]*

(c) **Prove** that any DFA whose language is L must have at least 375 states, using a fooling set argument.

Consider the recursive function Bond defined in the question handout.

(a) **Prove** that $|\text{Bond}(w)| \geq |w|$ for all strings w.

(b) **Prove** that $\text{Bond}(x \bullet y) = \text{Bond}(x) \bullet \text{Bond}(y)$ for all strings x and y.

Let L be the language $\left\{0^a 1^b 0^c \mid a = b \text{ or } a = c \text{ or } b = c\right\}$

1. **Prove** that L is *not* a regular language.

2. Describe a context-free grammar for L. You do not need to justify your answer.

For each statement below, check "Yes" if the statement is *always* true and check "No" otherwise, and write a *brief* (one short sentence) explanation of your answer. Read these statements very carefully—small details matter!

For any string $w \in (0 + 1)^*$, let w^C denote the *bitwise complement* of w, obtained by flipping every 0 in w to a 1, and vice versa. For example, $\varepsilon^C = \varepsilon$ and $000110^C = 111001$.

(a) If $2 + 2 = 5$, then zero is odd.

    [Yes]   [No]   _____

(b) $\{0^n1 \mid n > 0\}$ is the only infinite fooling set for the language $\{0^n10^n \mid n > 0\}$.

    [Yes]   [No]   _____

(c) $\{0^n10^n \mid n > 0\}$ is a context-free language.

    [Yes]   [No]   _____

(d) The context-free grammar $S \rightarrow 00S \mid S11 \mid 01$ generates the language $\{0^n1^n \mid n \geq 0\}$.

    [Yes]   [No]   _____

(e) Every regular language is recognized by a DFA with exactly one accepting state.

    [Yes]   [No]   _____

(f) Any language that can be decided by an NFA with ε-transitions can also be decided by an NFA without ε-transitions.

    [Yes]   [No]   _____

(g) If L is a regular language over the alphabet $\{0, 1\}$, then $\{xy^C \mid x, y \in L\}$ is also regular.

    [Yes]   [No]   _____

(h) If L is a regular language over the alphabet $\{0, 1\}$, then $\{ww^C \mid w \in L\}$ is also regular.

    [Yes]   [No]   _____

(i) The regular expression $(00 + 11)^*$ represents the language of all strings over $\{0, 1\}$ of even length.

    [Yes]   [No]   _____

(j) Let L_1, L_2 be two regular languages. The language $(L_1 + L_2)^*$ is also regular.

    [Yes]   [No]   _____

(scratch paper)

(scratch paper)

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