CS/ECE 374 A ✧ Fall 2025

Practice Final Exam 3

December 10, 2025

Name:	
NetID:	

- ***Don't panic!***
- You have 180 minutes to answer seven questions. The questions are described in more detail in a separate handout.
- If you brought anything except your writing implements, your two **hand-written** double-sided 8½" × 11" cheat sheets, and your university ID, please put it away for the duration of the exam. In particular, please turn off and put away *all* medically unnecessary electronic devices.
- Please clearly print your name and your NetID in the boxes above.
- Please also print your name at the top of every page of the answer booklet, except this cover page. We want to make sure that if a staple falls out, we can reassemble your answer booklet. (It doesn't happen often, but it does happen.)
- Greedy algorithms require formal proofs of correctness to receive any credit, even if they are correct. Otherwise, proofs or other justifications are required for full credit if and only if we explicitly ask for them, using the word ***prove*** or ***justify*** in bold italics.

- **Please do not write outside the black boxes on each page.** These indicate the area of the page that our scanners can actually scan. If the scanner can't see your work, we can't grade it.
- If you run out of space for an answer, please use the overflow/scratch pages at the back of the answer booklet, but **please clearly indicate where we should look**. If we can't find your work, we can't grade it.
- **Only work that is written into the stapled answer booklet will be graded**. In particular, you are welcome to detach scratch pages from the answer booklet, but any work on those detached pages will not be graded. We will provide additional scratch paper on request, but any work on that scratch paper will not be graded.
- Breathe in. Breathe out. You've got this.

CS/ECE 374 A ✧ Fall 2025 Practice Final 3 Problem 1	Name:

For each statement below, there are two boxes in the answer booklet labeled "Yes" and "No". Check "Yes" if the statement is ***always*** true and "No" otherwise, and give a ***brief*** (at most one short sentence) explanation of your answer. **Assume *P ≠ NP*.** If there is any other ambiguity or uncertainty about an answer, check "No".

Read each statement *very* carefully; some of these are deliberately subtle!

(a) Which of the following statements are true for ***every*** language $L \subseteq \{0, 1\}^*$?

- $(L^*)^*$ is infinite.

  Yes No ______________________________

- If L is decidable then its complement $\overline{L}$ is undecidable.

  Yes No ______________________________

- $\{\langle M \rangle \mid M \text{ accepts } L\}$ is undecidable.

  Yes No ______________________________

- Either L is finite or L is NP-hard.

  Yes No ______________________________

- Either L has an infinite fooling set or $L \in P$.

  Yes No ______________________________

Problem 1 continues onto the next page.

(b) Consider the following pair of languages:

- TREE = $\{G \mid G$ is a connected undirected graph with no cycles$\}$
- HAMPATH = $\{G \mid G$ is an undirected graph that contains a Hamiltonian path$\}$

(For concreteness, assume that in both of these languages, graphs are represented by their adjacency matrices.) Which of the following statements are true, assuming P $\neq$ NP?

- TREE is NP-hard.

  Yes No ______________________________

- TREE $\cap$ HAMPATH is NP-hard.

  Yes No ______________________________

- TREE $\cup$ HAMPATH is NP-hard.

  Yes No ______________________________

- HAMPATH is undecidable.

  Yes No ______________________________

- A reduction from TREE to HAMPATH would imply P = NP.

  Yes No ______________________________

CS/ECE 374 A ✧ Fall 2025 Practice Final 3 Problem 2	Name:

For each statement below, there are two boxes in the answer booklet labeled "Yes" and "No". Check "Yes" if the statement is ***always*** true and "No" otherwise, and give a ***brief*** (at most one short sentence) explanation of your answer. **Assume $P \neq NP$.** If there is any other ambiguity or uncertainty about an answer, check "No".

(a) Which of the following statements are true?

- The solution to the recurrence $T(n) = 3T(n/3) + O(n^2)$ is $T(n) = O(n^2 \log n)$.

  Yes No ____

- The solution to the recurrence $T(n) = T(n/2) + T(n/3) + T(n/6) + O(n)$ is $T(n) = O(n)$.

  Yes No ____

- There is a forest with 374 vertices and 225 edges. (Recall that a *forest* is an undirected graph with no cycles.)

  Yes No ____

- Given any directed graph G whose edges have positive weights, we can compute shortest paths from one vertex s to every other vertex of G in $O(VE)$ time using Bellman-Ford.

  Yes No ____

- Suppose $A[1..n]$ is an array of integers. Consider the following recursive function:

$$Ohio(i) = \begin{cases} 0 & \text{if } i < 1 \text{ or } i > n \\ A[i] + \max\left\{Ohio(i + A[i]),\ Ohio(i - A[i])\right\} & \text{otherwise} \end{cases}$$

  We can compute $Ohio(n)$ by memoizing this function into a two-dimensional array $Ohio[1..n]$, which we fill by increasing i in $O(n)$ time.

  Yes No ____

Problem 2 continues onto the next page.

(b) Suppose we want to prove that the following language is undecidable.

$$\text{CHALMERS} := \{\langle M\rangle \mid M \text{ accepts both } \texttt{STEAMED} \text{ and } \texttt{HAMS}\}$$

Professor Skinner suggests a reduction from the standard halting language

$$\text{HALT} := \{(\langle M\rangle, w) \mid M \text{ halts on inputs } w\}.$$

Specifically, Professor Skinner claims that if there is a Turing machine SUPERNINTENDO that decides the language CHALMERS, then the following algorithm decides HALT.

```
DECIDEHALT(⟨M⟩, w):
  Encode the following Turing machine:
      AURORABOREALIS(x):
        if x = STEAMED or x = HAMS or x = UTICA
            run M on input w
            return FALSE
        else
            return TRUE
  return SUPERNINTENDO(⟨AURORABOREALIS⟩)
```

Which of the following statements is true for all inputs $(\langle M\rangle, w)$?

- If M hangs on w, then AURORABOREALIS accepts ALBANY.

  Yes No ______

- If M accepts w, then SUPERNINTENDO accepts ⟨AURORABOREALIS⟩.

  Yes No ______

- If M hangs on w, then DECIDEHALT rejects $(\langle M\rangle, w)$.

  Yes No ______

- DECIDEHALT decides the language HALT. (That is, Professor Skinner's reduction is correct.)

  Yes No ______

- We could have proved that CHALMERS is undecidable using Rice's theorem instead of this reduction.

  Yes No ______

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Submit a solution to ***exactly one*** of the following problems.

(a) A *Hamiltonian bicycle* in a graph G is a pair of simple cycles in G, with identical lengths, such that every vertex of G lies on exactly one of the two cycles. ***Prove*** that it is NP-hard to determine whether a given graph G has a Hamiltonian bicycle.

(b) A *clique-partition* of a graph $G = (V, E)$ is a partition of V into disjoint subsets $V_1 \cup V_2 \cup \cdots \cup V_k$, such that for each index i, every pair of vertices in subset V_i is connected by an edge in G. The *size* of a clique partition is the number of subsets V_i. ***Prove*** that it is NP-hard to compute the minimum-size clique partition of a given undirected graph G.

In fact, both of these problems are NP-hard, but we only want a proof for one of them. Don't forget to tell us which problem you've chosen!

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Let T be a *full* binary tree, meaning that every node has either two children or no children.

- Recall that the *height* of a vertex v in T is the length of the longest path in T from v down to a leaf. In particular, every leaf of T has height zero.
- A vertex v is *AVL-balanced* if v is a leaf, or if the heights of v's children differ by at most 1. (You might recall from CS 225 that an ***AVL-tree*** is a binary search tree in which *every* vertex is AVL-balanced.)

Describe and analyze an algorithm to compute the number of AVL-balanced vertices in T.

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Practice Final 3 Problem 4	

Suppose we are given a directed graph $G = (V, E)$, where every edge $e \in E$ has a *positive* weight $w(e)$, along with two vertices s and t.

(a) Suppose each *vertex* of G is colored either orange, green, or purple. Describe and analyze an algorithm to find the shortest walk from s to t in G that never visits two consecutive *vertices* with the same color.

(b) Now suppose each *edge* of G is colored either orange, green, or purple. Describe and analyze an algorithm to find the shortest walk from s to t in G that never traverses two consecutive *edges* with the same color.

CS/ECE 374 A ✧ Fall 2025 Practice Final 3 Problem 5	Name:

(a) Describe and analyze an efficient algorithm to compute the maximum possible score for a game of Vankin's Mile, given the $n \times n$ array of values as input. *(See the question handout for a detailed description of Vankin's Mile.)*

(b) A variant called *Vankin's Niknav* adds an additional constraint: *The sequence of values that the token touches must be a* ***palindrome****.* Describe and analyze an efficient algorithm to compute the maximum possible score for an instance of Vankin's Niknav, given the $n \times n$ array of values as input.

CS/ECE 374 A ✧ Fall 2025 Practice Final 3 Problem 6	Name:

(a) Let L_a denote the set of all strings $w \in \{0, 1, 2\}^*$ such that $\#(1, w) + 2 \cdot \#(2, w)$ is divisible by 3. Describe a DFA or NFA that accepts L_a. (You do not need to prove that your answer is correct.)

(b) Let L_b denote the set of all strings $w \in \{0, 1, 2\}^*$ such that no two symbols appear the same number of times, or in other words, the integers $\#(0, w)$ and $\#(1, w)$ and $\#(2, w)$ are all different. ***Prove*** that L_b is not a regular language.

(overflow / scratch paper)

(overflow / scratch paper)

(overflow / scratch paper)

(overflow / scratch paper)

(overflow / scratch paper)

Some useful NP-hard problems. You are welcome to use any of these in your own NP-hardness proofs, except of course for the specific problem you are trying to prove NP-hard.

CircuitSat: Given a boolean circuit, are there any input values that make the circuit output True?

3Sat: Given a boolean formula in conjunctive normal form, with exactly three distinct literals per clause, does the formula have a satisfying assignment?

MaxIndependentSet: Given an undirected graph G, what is the size of the largest subset of vertices in G that have no edges among them?

MaxClique: Given an undirected graph G, what is the size of the largest complete subgraph of G?

MinVertexCover: Given an undirected graph G, what is the size of the smallest subset of vertices that touch every edge in G?

MinSetCover: Given a collection of subsets $S_1, S_2, \ldots, S_m$ of a set S, what is the size of the smallest subcollection whose union is S?

MinHittingSet: Given a collection of subsets $S_1, S_2, \ldots, S_m$ of a set S, what is the size of the smallest subset of S that intersects every subset S_i?

3Color: Given an undirected graph G, can its vertices be colored with three colors, so that every edge touches vertices with two different colors?

ChromaticNumber: Given an undirected graph G, what is the minimum number of colors required to color its vertices, so that every edge touches vertices with two different colors?

HamiltonianPath: Given graph G (either directed or undirected), is there a path in G that visits every vertex exactly once?

HamiltonianCycle: Given a graph G (either directed or undirected), is there a cycle in G that visits every vertex exactly once?

TravelingSalesman: Given a graph G (either directed or undirected) with weighted edges, what is the minimum total weight of any Hamiltonian path/cycle in G?

LongestPath: Given a graph G (either directed or undirected, possibly with weighted edges), what is the length of the longest simple path in G?

SteinerTree: Given an undirected graph G with some of the vertices marked, what is the minimum number of edges in a subtree of G that contains every marked vertex?

SubsetSum: Given a set X of positive integers and an integer k, does X have a subset whose elements sum to k?

Partition: Given a set X of positive integers, can X be partitioned into two subsets with the same sum?

3Partition: Given a set X of $3n$ positive integers, can X be partitioned into n three-element subsets, all with the same sum?

IntegerLinearProgramming: Given a matrix $A \in \mathbb{Z}^{n \times d}$ and two vectors $b \in \mathbb{Z}^n$ and $c \in Z^d$, compute $\max\{c \cdot x \mid Ax \leq b, x \geq 0, x \in \mathbb{Z}^d\}$.

FeasibleILP: Given a matrix $A \in \mathbb{Z}^{n \times d}$ and a vector $b \in \mathbb{Z}^n$, determine whether the set of feasible integer points $\max\{x \in \mathbb{Z}^d \mid Ax \leq b, x \geq 0\}$ is empty.

Draughts: Given an $n \times n$ international draughts configuration, what is the largest number of pieces that can (and therefore must) be captured in a single move?

SuperMarioBrothers: Given an $n \times n$ Super Mario Brothers level, can Mario reach the castle?