

CS/ECE 374 A ✧ Fall 2025  
☞ Practice Final Exam 1 ☞  
December 7, 2023

Name:	
NetID:	

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- ***Don't panic!***
  - You have 180 minutes to answer seven questions. The questions are described in more detail in a separate handout.
  - If you brought anything except your writing implements, your two **hand-written** double-sided  $8\frac{1}{2}'' \times 11''$  cheat sheets, and your university ID, please put it away for the duration of the exam. In particular, please turn off and put away *all* medically unnecessary electronic devices.
  - Please clearly print your name and your NetID in the boxes above.
  - Please also print your name at the top of every page of the answer booklet, except this cover page. We want to make sure that if a staple falls out, we can reassemble your answer booklet. (It doesn't happen often, but it does happen.)
  - Greedy algorithms require formal proofs of correctness to receive any credit, even if they are correct. Otherwise, proofs or other justifications are required for full credit if and only if we explicitly ask for them, using the word ***prove*** or ***justify*** in bold italics.

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- **Please do not write outside the black boxes on each page.** These indicate the area of the page that our scanners can actually scan. If the scanner can't see your work, we can't grade it.
  - If you run out of space for an answer, please use the overflow/scratch pages at the back of the answer booklet, but **please clearly indicate where we should look**. If we can't find your work, we can't grade it.
  - **Only work that is written into the stapled answer booklet will be graded.** In particular, you are welcome to detach scratch pages from the answer booklet, but any work on those detached pages will not be graded. We will provide additional scratch paper on request, but any work on that scratch paper will not be graded.
  - Breathe in. Breathe out. You've got this.
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(a) Which of the following statements are true for *at least one* language  $L \subseteq \{0, 1\}^*$ ?

- $L^* = (L^*)^*$

Yes

No

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- $L$  is decidable, but  $L^*$  is undecidable.

Yes

No

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- $L$  is neither regular nor NP-hard.

Yes

No

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- $L$  is in P, and  $L$  has an infinite fooling set.

Yes

No

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- The language  $\{\langle M \rangle \mid M \text{ accepts } L\}$  is undecidable.

Yes

No

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(b) Suppose there is a *polynomial-time* reduction from some language  $A$  over the alphabet  $\{0, 1\}$  to some other language  $B$  over the alphabet  $\{0, 1\}$ . Which of the following statements are *always* true, assuming  $P \neq NP$ ?

- $A$  is a subset of  $B$ .

Yes

No

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- If  $B \in P$ , then  $A \in P$ .

Yes

No

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- If  $B$  is NP-hard, then  $A$  is NP-hard.

Yes

No

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- If  $B$  is regular, then  $A$  is regular.

Yes

No

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- If  $B$  is regular, then  $A$  is decidable.

Yes

No

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(a) For each statement below, check “YES” if the statement is *always* true and “NO” otherwise, and give a *brief* (at most one short sentence) explanation of your answer. **Assume  $P \neq NP$ .** If there is any other ambiguity or uncertainty about an answer, write “NO”.

- The solution to the recurrence  $T(n) = 8T(n/2) + O(n^2)$  is  $T(n) = O(n^2)$ .

Yes	No
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- The solution to the recurrence  $T(n) = 2T(n/8) + O(n^2)$  is  $T(n) = O(n^2)$ .

Yes	No
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- Every directed acyclic graph contains at least one sink.

Yes	No
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- Given *any* undirected graph  $G = (V, E)$ , we can compute a spanning tree of  $G$  in  $O(V + E)$  time using whatever-first search.

Yes	No
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- Suppose  $A[1..n]$  is an array of integers. Consider the following recursive function:

$$What(i, j) = \begin{cases} 0 & \text{if } i < 0 \text{ or } i > n \\ 0 & \text{if } j < 0 \text{ or } j > n \\ \max \left\{ \begin{array}{l} What(i, j-1) \\ What(i-1, j) \\ A[i] \cdot A[j] + What(i+1, j+1) \end{array} \right\} & \text{otherwise} \end{cases}$$

We can memoize this function into an array  $What[0..n, 0..n]$  in  $O(n^2)$  time, by increasing  $i$  in the outer loop and increasing  $j$  in the inner loop.

Yes	No
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Problem 2 continues on the next page.

(b) Consider the following pair of languages:

- $\text{DIRHAMPATH} := \{G \mid G \text{ is a directed graph with a Hamiltonian path}\}$
- $\text{ACYCLIC} := \{G \mid G \text{ is a directed acyclic graph}\}$

(For concreteness, assume that in both of these languages, graphs are represented by their adjacency matrices.) Which of the following statements are true, assuming  $P \neq \text{NP}$ ?

- $\text{ACYCLIC} \in \text{NP}$

Yes

No

- $\text{ACYCLIC} \cap \text{DIRHAMPATH} \in \text{P}$

Yes

No

- $\text{DIRHAMPATH}$  is decidable.

Yes

No

- A polynomial-time reduction from  $\text{DIRHAMPATH}$  to  $\text{ACYCLIC}$  would imply  $\text{P}=\text{NP}$ .

Yes

No

- A polynomial-time reduction from  $\text{ACYCLIC}$  to  $\text{DIRHAMPATH}$  would imply  $\text{P}=\text{NP}$ .

Yes

No

Describe and analyze an algorithm to determine whether the language accepted by a given DFA is finite or infinite. You can assume the input alphabet of the DFA is  $\{0, 1\}$ . *[Hint: DFAs are directed graphs.]*

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Suppose you are asked to tile a  $2 \times n$  grid of squares with dominos ( $1 \times 2$  rectangles). Each domino must cover exactly two grid squares, either horizontally or vertically, and each grid square must be covered by exactly one domino.

Each grid square is worth some number of points, which could be positive, negative, or zero. The **value** of a domino tiling is the sum of the points in squares covered by vertical dominos, **minus** the sum of the points in squares covered by horizontal dominos.

Describe an algorithm to compute the largest possible value of a domino tiling of a given  $2 \times n$  grid. Your input is an array *Points*[1 .. 2, 1 .. *n*] of point values.

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*Prove* that the following problem (which we call MATCH) is NP-hard. The input is a finite set  $S$  of strings, all of the same length  $n$ , over the alphabet  $\{0, 1, 2\}$ . The problem is to determine whether there is a string  $w \in \{0, 1\}^n$  such that for every string  $s \in S$ , the strings  $s$  and  $w$  have the same symbol in at least one position.

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Suppose you are given a height map of a mountain, in the form of an  $n \times n$  grid of evenly spaced points, each labeled with an elevation value. You can safely hike directly from any point to any neighbor immediately north, south, east, or west, but only if the elevations of those two points differ by at most  $\Delta$ . (The value of  $\Delta$  depends on your hiking experience and your physical condition.)

Describe and analyze an algorithm to determine the longest hike from some point  $s$  to some other point  $t$ , where the hike consists of an uphill climb (where elevations must increase at each step) followed by a downhill climb (where elevations must decrease at each step). Your input consists of an array  $Elevation[1..n, 1..n]$  of elevation values, the starting point  $s$ , the target point  $t$ , and the parameter  $\Delta$ .

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- (a) Let  $L_a$  denote the set of all strings in  $\{0, 1\}^*$  where every  $0$  is followed immediately by at least one  $1$ . Describe a DFA or NFA that accepts  $L_a$  **and** give a regular expression that describes  $L_a$ . (You do not need to prove that your answers are correct.)
- (b) Let  $L_b$  denote the set of all strings in  $\{0, 1\}^*$  whose run lengths are increasing; that is, every run except the last is followed immediately by a *longer* run. **Prove** that  $L_b$  is not a regular language.
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(overflow / scratch paper)

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**Some useful NP-hard problems.** You are welcome to use any of these in your own NP-hardness proofs, except of course for the specific problem you are trying to prove NP-hard.

**CIRCUITSAT:** Given a boolean circuit, are there any input values that make the circuit output TRUE?

**3SAT:** Given a boolean formula in conjunctive normal form, with exactly three distinct literals per clause, does the formula have a satisfying assignment?

**MAXINDEPENDENTSET:** Given an undirected graph  $G$ , what is the size of the largest subset of vertices in  $G$  that have no edges among them?

**MAXCLIQUE:** Given an undirected graph  $G$ , what is the size of the largest complete subgraph of  $G$ ?

**MINVERTEXCOVER:** Given an undirected graph  $G$ , what is the size of the smallest subset of vertices that touch every edge in  $G$ ?

**MINSETCOVER:** Given a collection of subsets  $S_1, S_2, \dots, S_m$  of a set  $S$ , what is the size of the smallest subcollection whose union is  $S$ ?

**MINHITTINGSET:** Given a collection of subsets  $S_1, S_2, \dots, S_m$  of a set  $S$ , what is the size of the smallest subset of  $S$  that intersects every subset  $S_i$ ?

**3COLOR:** Given an undirected graph  $G$ , can its vertices be colored with three colors, so that every edge touches vertices with two different colors?

**CHROMATICNUMBER:** Given an undirected graph  $G$ , what is the minimum number of colors required to color its vertices, so that every edge touches vertices with two different colors?

**HAMILTONIANPATH:** Given graph  $G$  (either directed or undirected), is there a path in  $G$  that visits every vertex exactly once?

**HAMILTONIANCYCLE:** Given a graph  $G$  (either directed or undirected), is there a cycle in  $G$  that visits every vertex exactly once?

**TRAVELINGSALESMAN:** Given a graph  $G$  (either directed or undirected) with weighted edges, what is the minimum total weight of any Hamiltonian path/cycle in  $G$ ?

**LONGESTPATH:** Given a graph  $G$  (either directed or undirected, possibly with weighted edges), what is the length of the longest simple path in  $G$ ?

**STEINERTREE:** Given an undirected graph  $G$  with some of the vertices marked, what is the minimum number of edges in a subtree of  $G$  that contains every marked vertex?

**SUBSETSUM:** Given a set  $X$  of positive integers and an integer  $k$ , does  $X$  have a subset whose elements sum to  $k$ ?

**PARTITION:** Given a set  $X$  of positive integers, can  $X$  be partitioned into two subsets with the same sum?

**3PARTITION:** Given a set  $X$  of  $3n$  positive integers, can  $X$  be partitioned into  $n$  three-element subsets, all with the same sum?

**INTEGERLINEARPROGRAMMING:** Given a matrix  $A \in \mathbb{Z}^{n \times d}$  and two vectors  $b \in \mathbb{Z}^n$  and  $c \in \mathbb{Z}^d$ , compute  $\max\{c \cdot x \mid Ax \leq b, x \geq 0, x \in \mathbb{Z}^d\}$ .

**FEASIBLEILP:** Given a matrix  $A \in \mathbb{Z}^{n \times d}$  and a vector  $b \in \mathbb{Z}^n$ , determine whether the set of feasible integer points  $\max\{x \in \mathbb{Z}^d \mid Ax \leq b, x \geq 0\}$  is empty.

**DRAUGHTS:** Given an  $n \times n$  international draughts configuration, what is the largest number of pieces that can (and therefore must) be captured in a single move?

**SUPERMARIOBROTHERS:** Given an  $n \times n$  Super Mario Brothers level, can Mario reach the castle?