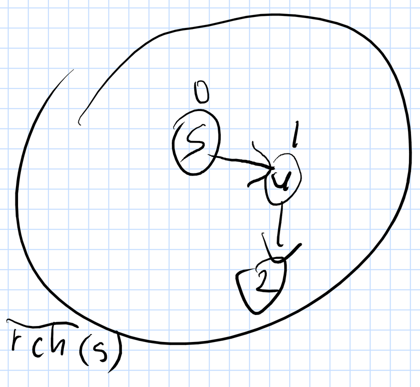
L17: 10/24/2024

rch(s)

$\overline{rch}(s)$



$rch(s) \cap \overline{rch}(s)$

$\shortparallel$

scc(s)

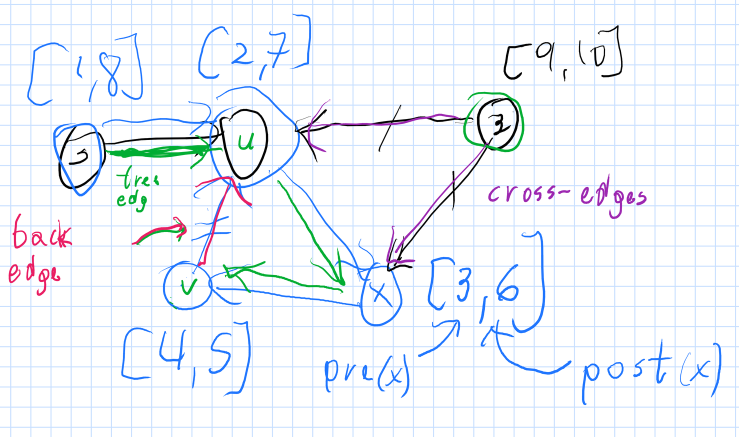
DFS: Depth first search

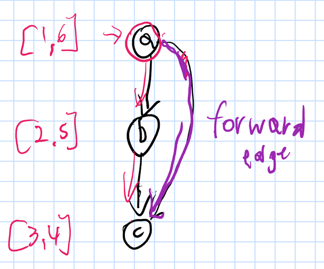
$V(G) = \{1, \dots, n\}$

DFS(G):
  visited[1,...,n] ← 0
  T ← ∅
  t ← 0
  for i = 1,...,n
    if not visited[i]
      DFS(i)

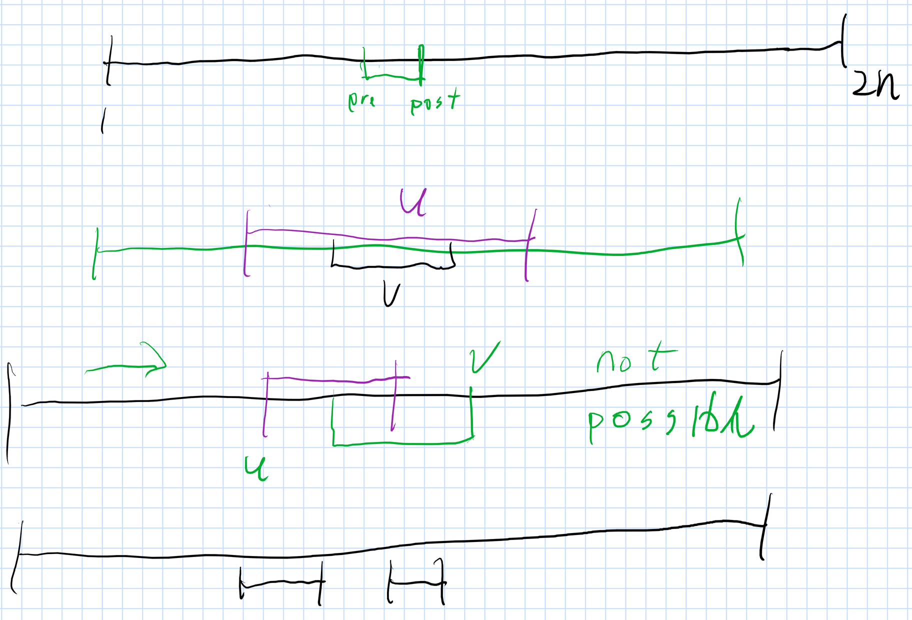
DFS(v):
  visited[v] ← TRUE
  pre[v] ← ++t
  for v → x ∈ Out(v)
    if not visited[x]
      add v → x to T
      DFS(x)
  post[v] ← ++t.

$O(n+m)$





$I(v) = [pre(v), post(v)]$

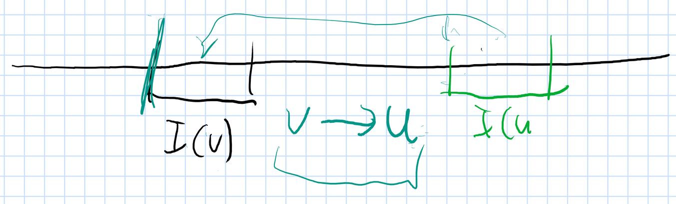


v was visited during DFS(u)

– tree edges (marked by DFS)

– $u \to v$ forward edge

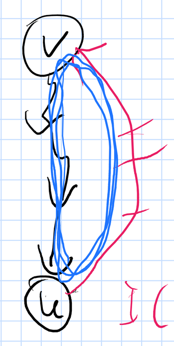
iff $I(u) \supseteq I(v)$



– $u \to v$ backward edge

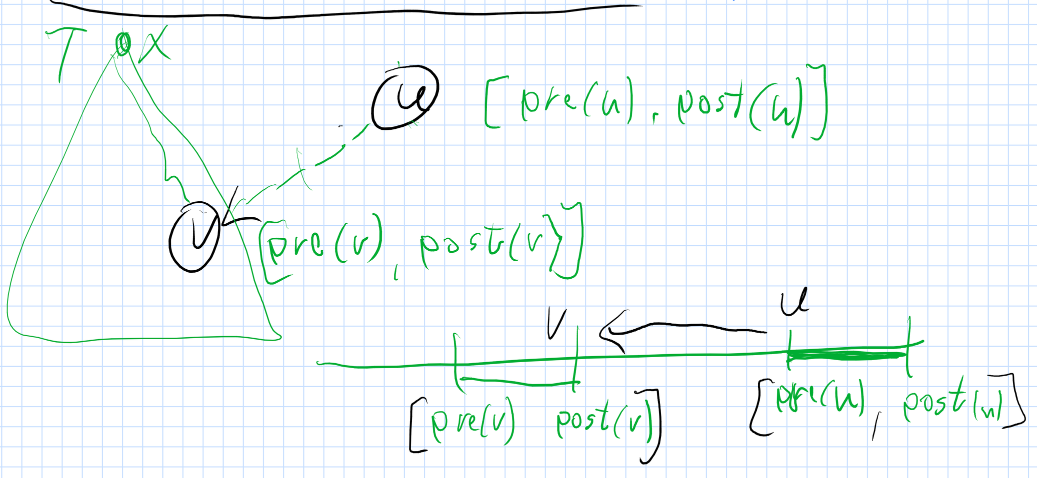
$I(u) \subseteq I(v)$

– $u \to v$ cross edge



$I(u) \subseteq I(v)$

if a graph has a backward edge then it contains a cycle.



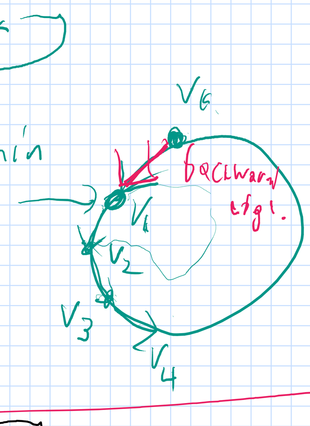
Claim G has a cycle ⟺ ∃ backward edge in the DFS.

proof

∃C cycle in G ⇒ ∃ backward edge.

$C = v_1 \to v_2 \to v_3 \to \dots \to v_k$

v_1 = vertex in G with min pre number among all vertices in G.

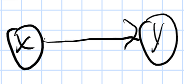


G : DAG

partial order

no cycles

$\boxed{x \prec y}$

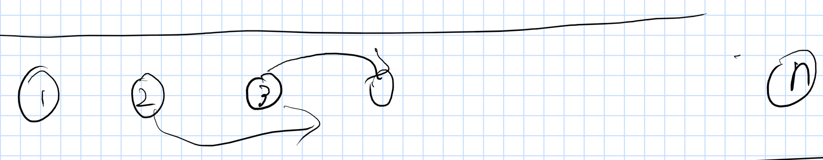


$\boxed{x<y \quad y<z \quad z<x}$

not allowed

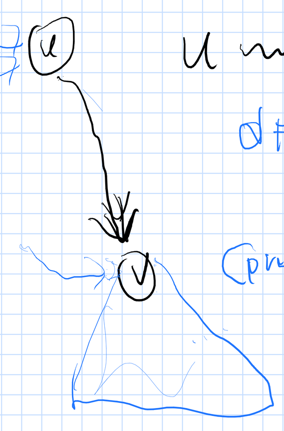
Topological ordering

Given partial order $\prec$ on a set S of n elements, its a numbering of $S = \{s_1, s_2, \dots, s_n\}$ s.t. $\forall s_i \prec s_j \Rightarrow i < j$.



Do DFS on the graph.

sort the vertices of the graph by decreasing post numbers.



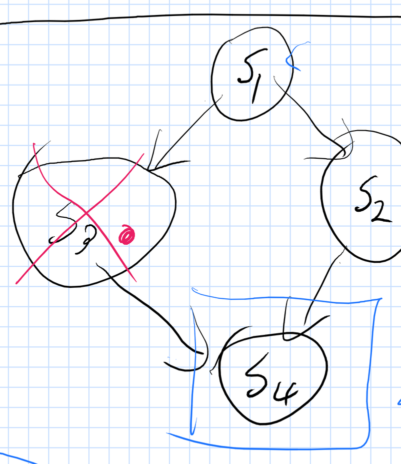
$u \rightsquigarrow v \Rightarrow post(u) > post(v)$

dfs(v) is inside dfs(u)

[pre(v), post(v)]

$O(n+m)$

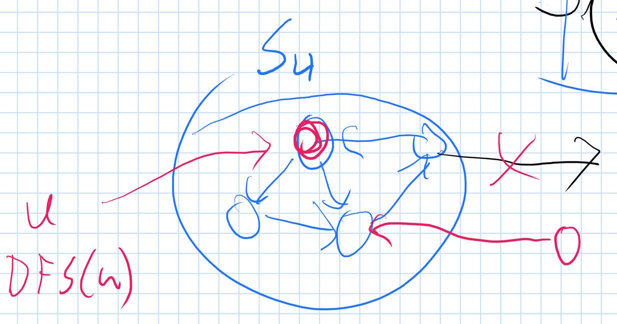
$O(n(n+m))$



meta graph

G^{SCC} is a DAG

$O(n+m)$

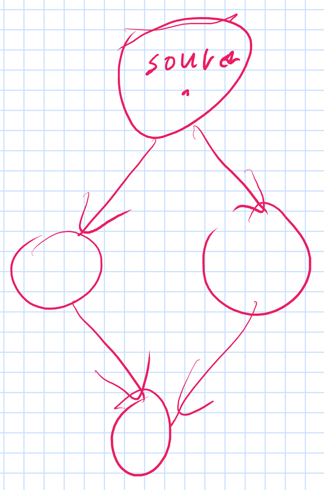


G^{rev}

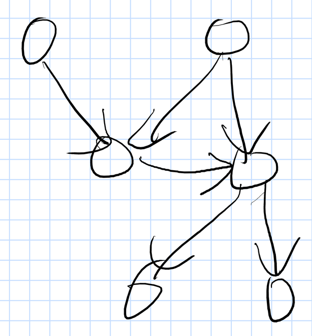
$(G^{rev})^{SCC}$

G^{rev}

DFS compute the pre post numbering



Take the vertex with max post numbering.



The vertex v with maximum post numbering is a source vertex.

Claim The vertex with max post numbering in G^{rev} belongs to a source SCC of G^{rev}.