23.1.3
Other NP Complete Problems

Proving that a problem **X** is **NP-Complete**

To prove ***X*** is **NP-Complete**, show

1. Show that ***X*** is in **NP**.
2. Give a polynomial-time reduction from a known **NP-Complete** problem such as **SAT** to ***X***

SAT $\leq_P$ ***X*** implies that every **NP** problem ***Y*** $\leq_P$ ***X***. Why?
Transitivity of reductions:

Y $\leq_P$ ***SAT*** and ***SAT*** $\leq_P$ ***X*** and hence ***Y*** $\leq_P$ ***X***.

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3-SAT is NP-Complete

- **3-SAT** is in ***NP***
- **SAT** $\leq_P$ **3-SAT** as we saw

NP-Completeness via Reductions

1. **SAT** is **NP-Complete** due to Cook-Levin theorem
2. **SAT** $\leq_P$ **3-SAT**
3. **3-SAT** $\leq_P$ **Independent Set**
4. **Independent Set** $\leq_P$ **Vertex Cover**
5. **Independent Set** $\leq_P$ **Clique**
6. **3-SAT** $\leq_P$ **3-Color**
7. **3-SAT** $\leq_P$ **Hamiltonian Cycle**

Hundreds and thousands of different problems from many areas of science and engineering have been shown to be **NP-Complete**.

A surprisingly frequent phenomenon!

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THE END

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(for now)