18.1.3
Restating problem of Shortest path with negative edges

Alternatively: Finding Shortest Walks

Given a graph $G = (V, E)$:

1. A path is a sequence of distinct vertices $v_1, v_2, \ldots, v_k$ such that $(v_i, v_{i+1}) \in E$ for $1 \le i \le k - 1$.
2. A walk is a sequence of vertices $v_1, v_2, \ldots, v_k$ such that $(v_i, v_{i+1}) \in E$ for $1 \le i \le k - 1$. Vertices are allowed to repeat.

Define $dist(u, v)$ to be the length of a shortest walk from u to v.

1. If there is a walk from u to v that contains negative length cycle then $dist(u, v) = -\infty$
2. Else there is a path with at most $n - 1$ edges whose length is equal to the length of a shortest walk and $dist(u, v)$ is finite

Helpful to think about walks

Shortest Paths with Negative Edge Lengths

Algorithmic Problems

Input: A directed graph $G = (V, E)$ with edge lengths (could be negative). For edge $e = (u, v)$, $\ell(e) = \ell(u, v)$ is its length.

Questions:

1. Given nodes s, t, either find a negative length cycle C that s can reach or find a shortest path from s to t.
2. Given node s, either find a negative length cycle C that s can reach or find shortest path distances from s to all reachable nodes.
3. Check if G has a negative length cycle or not.

Shortest Paths with Negative Edge Lengths

In Undirected Graphs

Note: With negative lengths, shortest path problems and negative cycle detection in undirected graphs cannot be reduced to directed graphs by bi-directing each undirected edge. Why?

Problem can be solved efficiently in undirected graphs but algorithms are different and significantly more involved than those for directed graphs. One need to compute $\boldsymbol{T}$-joins in the relevant graph. Pretty painful stuff.

THE END

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(for now)