17.4.2
Variants on the shortest path problem

Shortest paths between sets of nodes

Suppose we are given $\boldsymbol{S} \subset \boldsymbol{V}$ and $\boldsymbol{T} \subset \boldsymbol{V}$. Want to find shortest path from $\boldsymbol{S}$ to $\boldsymbol{T}$ defined as:

$$\mathbf{dist}(\boldsymbol{S}, \boldsymbol{T}) = \min_{\boldsymbol{s} \in \boldsymbol{S}, \boldsymbol{t} \in \boldsymbol{T}} \mathbf{dist}(\boldsymbol{s}, \boldsymbol{t})$$

How do we find $\mathbf{dist}(\boldsymbol{S}, \boldsymbol{T})$?

Example Problem

You want to go from your house to a friend's house. Need to pick up some dessert along the way and hence need to stop at one of the many potential stores along the way. How do you calculate the "shortest" trip if you include this stop?

Given $G = (V, E)$ and edge lengths $\ell(e), e \in E$. Want to go from s to t. A subset $X \subset V$ that corresponds to stores. Want to find $\min_{x \in X} d(s, x) + d(x, t)$.

Basic solution: Compute for each $x \in X$, $d(s, x)$ and $d(x, t)$ and take minimum. $2|X|$ shortest path computations. $O(|X|(m + n \log n))$.

Better solution: Compute shortest path distances from s to every node $v \in V$ with one Dijkstra. Compute from every node $v \in V$ shortest path distance to t with one Dijkstra.

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THE END

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(for now)