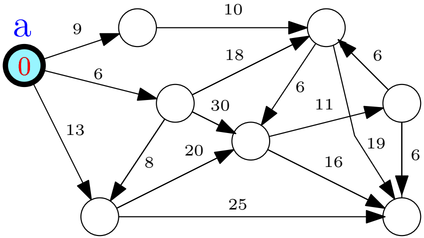
17.3.7
Dijkstra's algorithm

Example: Dijkstra algorithm in action



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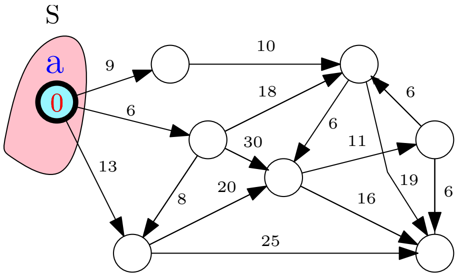

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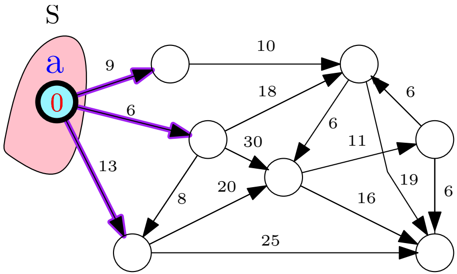

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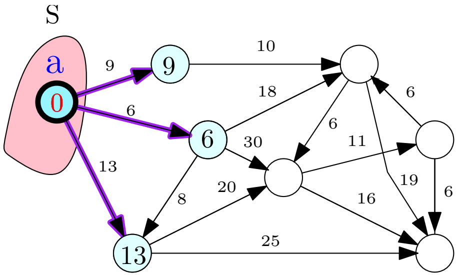

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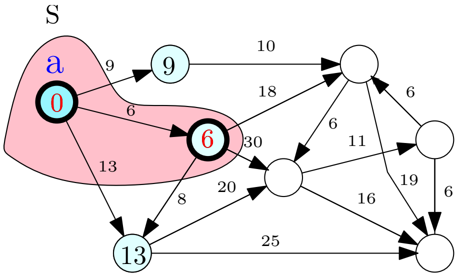

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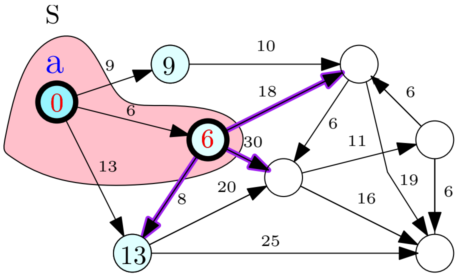

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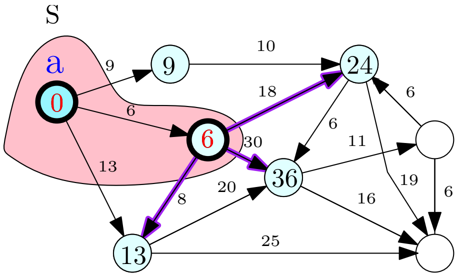

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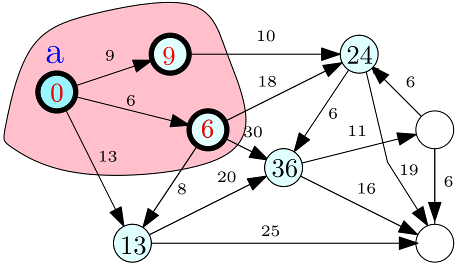

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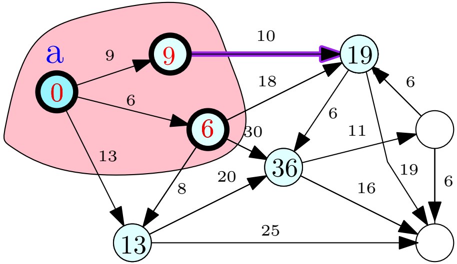

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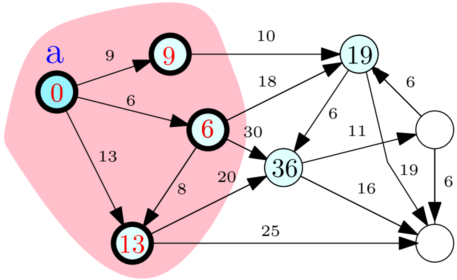

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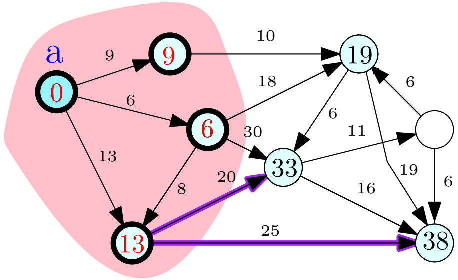

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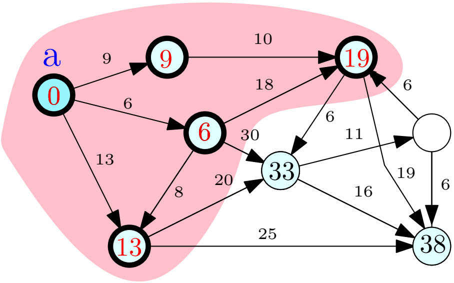

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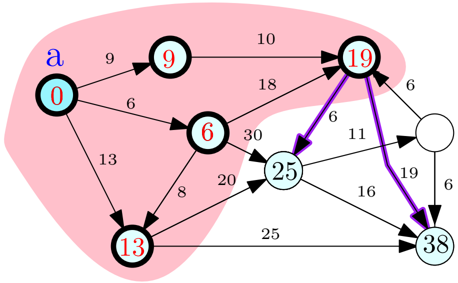

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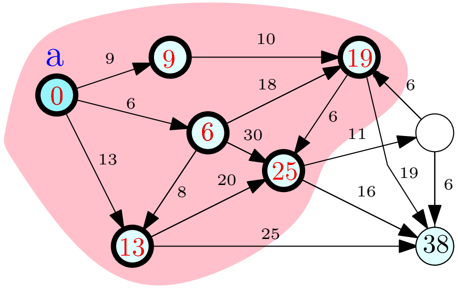

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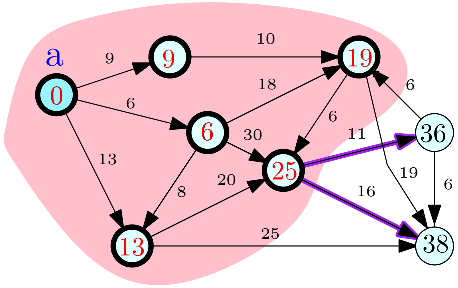

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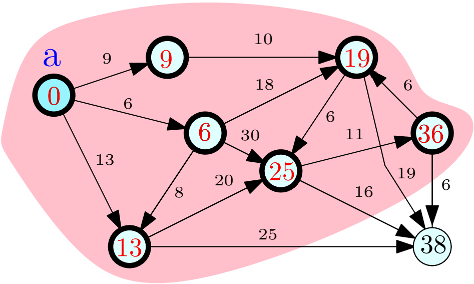

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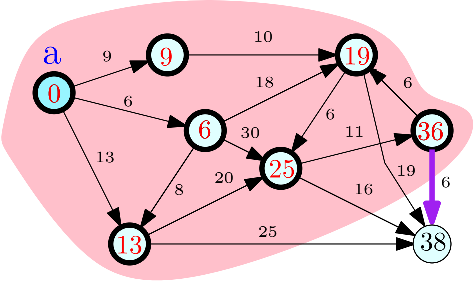

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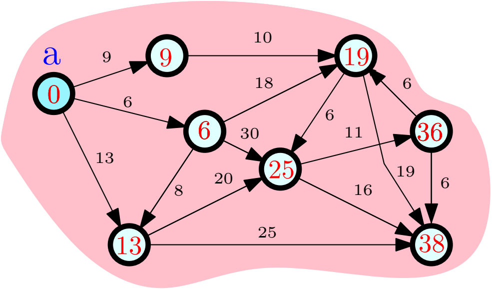

Improved Algorithm

1. Main work is to compute the $d'(s,u)$ values in each iteration
2. $d'(s,u)$ changes from iteration i to $i+1$ only because of the node v that is added to X in iteration i.

```
Initialize for each node v, dist(s,v) = d'(s,v) = ∞
Initialize X = ∅, d'(s,s) = 0
for i = 1 to |V| do
    // X contains the i − 1 closest nodes to s,
    //        and the values of d'(s,u) are current
    Let v be node realizing d'(s,v) = min_{u∈V−X} d'(s,u)
    dist(s,v) = d'(s,v)
    X = X ∪ {v}
    Update d'(s,u) for each u in V − X as follows:
        d'(s,u) = min(d'(s,u), dist(s,v) + ℓ(v,u))
```

Running time: $O(m+n^2)$ time.

1. n outer iterations and in each iteration following steps
2. updating $d'(s,u)$ after v is added takes $O(deg(v))$ time so total work is $O(m)$

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Running time: $O(m + n^2)$ time.

1. n outer iterations and in each iteration following steps
2. updating $d'(s, u)$ after v is added takes $O(deg(v))$ time so total work is $O(m)$ since a node enters X only once
3. Finding v from $d'(s, u)$ values is $O(n)$ time

Dijkstra's Algorithm

1. eliminate $d'(s, u)$ and let $\text{dist}(s, u)$ maintain it
2. update *dist* values after adding v by scanning edges out of v

```
Initialize for each node v, dist(s, v) = ∞
Initialize X = ∅, dist(s, s) = 0
for i = 1 to |V| do
    Let v be such that dist(s, v) = min_{u∈V−X} dist(s, u)
    X = X ∪ {v}
    for each u in Adj(v) do
        dist(s, u) = min(dist(s, u), dist(s, v) + ℓ(v, u))
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Priority Queues to maintain *dist* values for faster running time

1. Using heaps and standard priority queues: $O((m + n) \log n)$
2. Using Fibonacci heaps: $O(m + n \log n)$.

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THE END

...

(for now)