17.3.5
The basic algorithm: Find the ith closest vertex

A Basic Strategy

Explore vertices in increasing order of distance from s:
(For simplicity assume that nodes are at different distances from s and that no edge has zero length)

```
Initialize for each node v, dist(s, v) = ∞
Initialize X = {s},
for i = 2 to |V| do
    (* Invariant: X contains the i − 1 closest nodes to s *)
    Among nodes in V − X, find the node v that is the
            ith closest to s
    Update dist(s, v)
    X = X ∪ {v}
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How can we implement the step in the for loop?

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Finding the **i**th closest node

1. X contains the $i - 1$ closest nodes to s
2. Want to find the ith closest node from $V - X$.

What do we know about the ith closest node?

Claim

Let P be a shortest path from s to v where v is the ith closest node. Then, all intermediate nodes in P belong to X.

Proof.

If P had an intermediate node u not in X then u will be closer to s than v. Implies v is not the ith closest node to s - recall that X already has the $i - 1$ closest nodes. □

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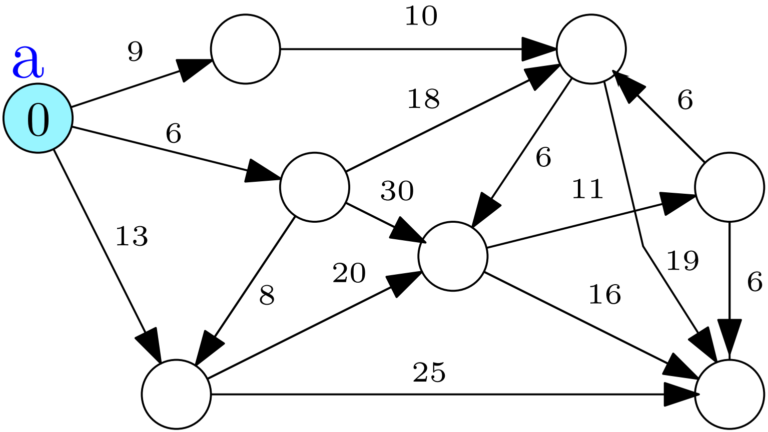
Finding the **i**th closest node repeatedly

An example



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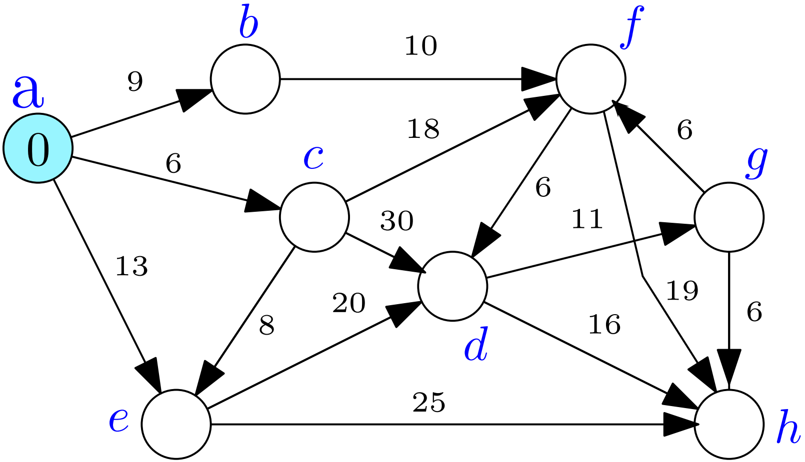

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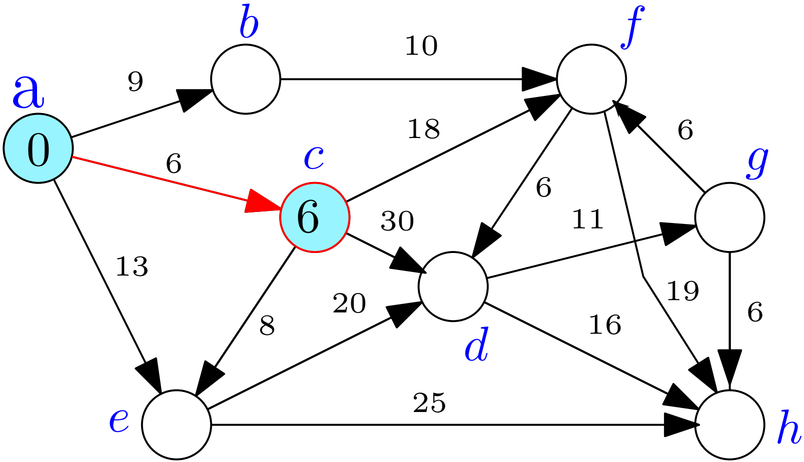

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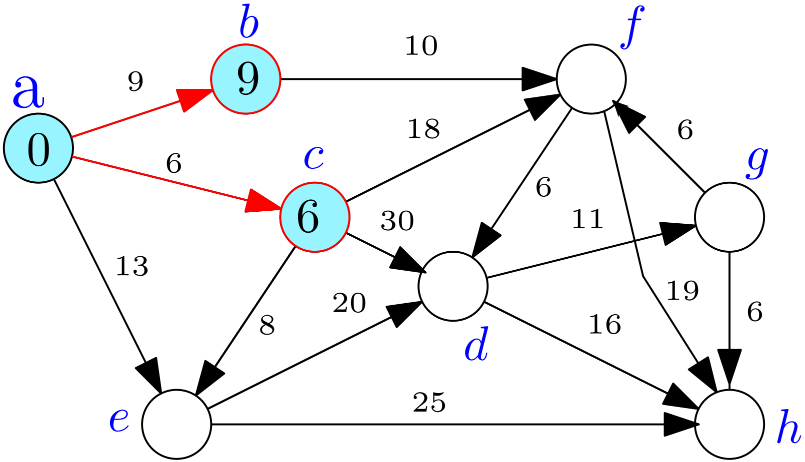

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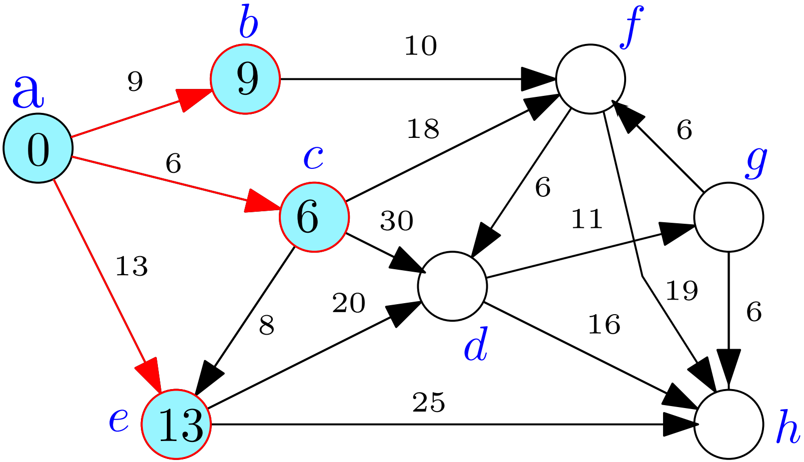

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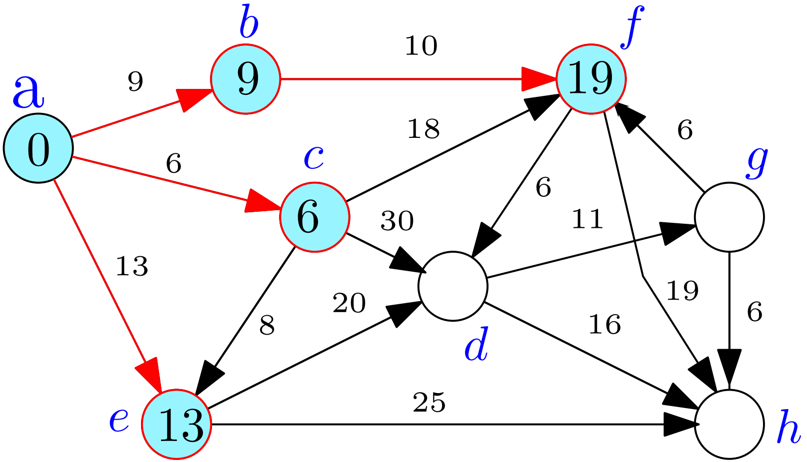

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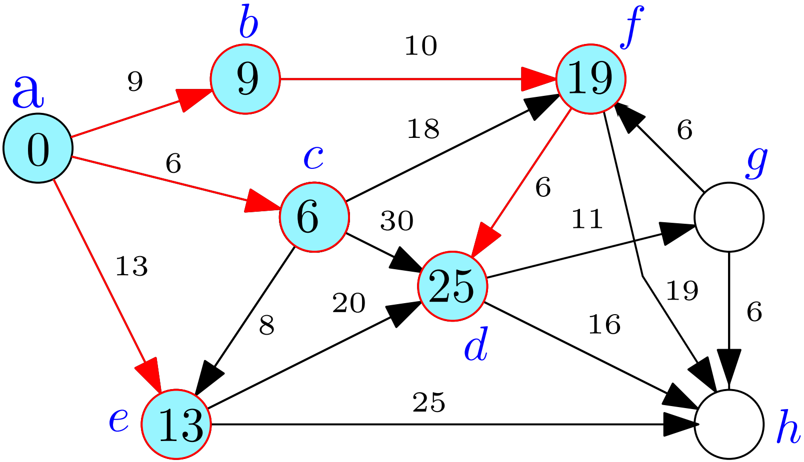

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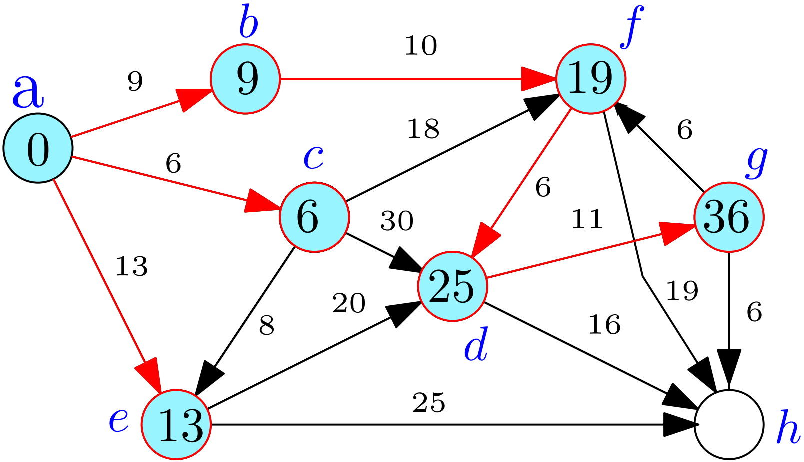

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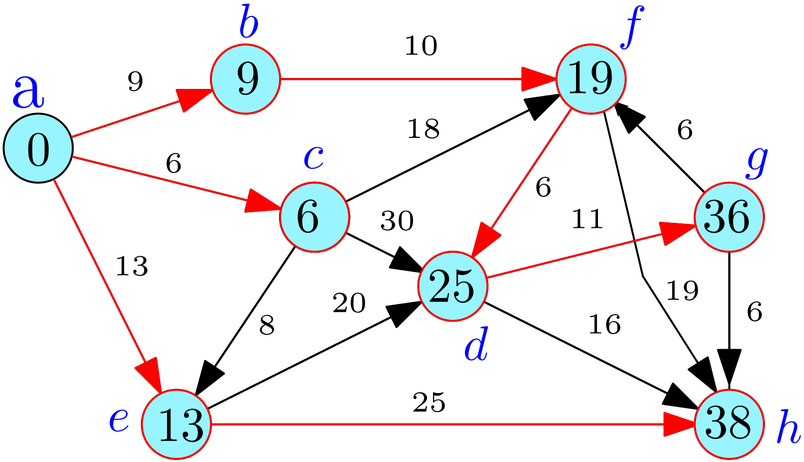

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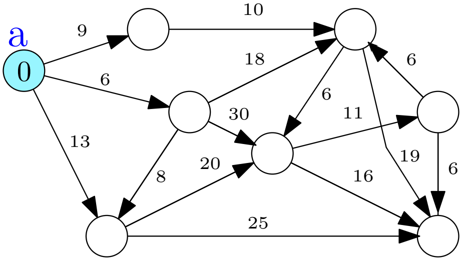

Corollary

The ***i*** *th closest node is adjacent to* ***X***.

Summary

Proved that the basic algorithm is (intuitively) correct...

...but is missing details

...and how to implement efficiently?

THE END

...

(for now)