

## 16.3

### Depth First Search (DFS)

## **16.3.1**

# Depth First Search (DFS) in Undirected Graphs

# Depth First Search

- ① **DFS** special case of Basic Search.
- ② **DFS** is useful in understanding graph structure.
- ③ **DFS** used to obtain linear time ( $O(m + n)$ ) algorithms for
  - ① Finding cut-edges and cut-vertices of undirected graphs
  - ② Finding strong connected components of directed graphs
- ④ ...many other applications as well.

# DFS in Undirected Graphs

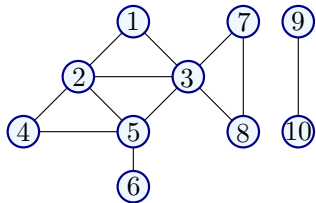
Recursive version. Easier to understand some properties.

```
DFS( $G$ )  
  for all  $u \in V(G)$  do  
    Mark  $u$  as unvisited  
    Set  $\text{pred}(u)$  to null  
     $T$  is set to  $\emptyset$   
    while  $\exists$  unvisited  $u$  do  
      DFS( $u$ )  
  Output  $T$ 
```

```
DFS( $u$ )  
  Mark  $u$  as visited  
  for each  $uv$  in  $\text{Out}(u)$  do  
    if  $v$  is not visited then  
      add edge  $uv$  to  $T$   
      set  $\text{pred}(v)$  to  $u$   
      DFS( $v$ )
```

Implemented using a global array *Visited* for all recursive calls.  
 $T$  is the search tree/forest.

## Example



Edges classified into two types:  $uv \in E$  is a

- 1 **tree edge**: belongs to  $T$
- 2 **non-tree edge**: does not belong to  $T$

# Properties of DFS tree

## Proposition

- 1  $T$  is a forest
- 2 connected components of  $T$  are same as those of  $G$ .
- 3 If  $uv \in E$  is a non-tree edge then, in  $T$ , either:
  - 1  $u$  is an ancestor of  $v$ , or
  - 2  $v$  is an ancestor of  $u$ .

**Question:** Why are there no cross-edges?

## Exercise

Prove that **DFS** of a graph  $G$  with  $n$  vertices and  $m$  edges takes  $O(n + m)$  time.

**THE END**

...

**(for now)**