

13.3

Checking if a string is in L^*

L^*

Problem

Input A string $w \in \Sigma^*$ and access to a language $L \subseteq \Sigma^*$ via function **IsInL**(*string* x) that decides whether x is in L

Goal Decide if $w \in L^*$ using **IsInL**(*string* x) as a black box sub-routine

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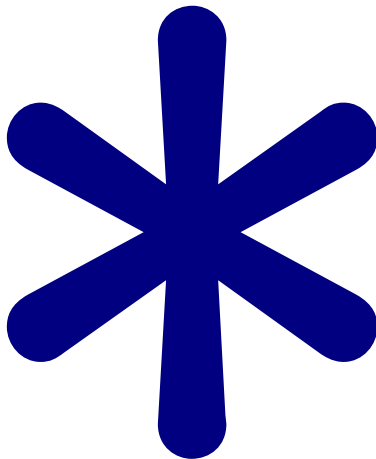


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Example 13.1.

Suppose L is *English* and we have a procedure to check whether a string/word is in the *English* dictionary.

- Is the string “isthisanenglishsentence” in *English*?
- Is “stampstamp” in *English*?
- Is “zibzzzad” in *English*?

Recursive Solution

When is $w \in L^*$?

$w \in L^* \iff w \in L$ or if $w = uv$ where $u \in L^*$ and $v \in L, |v| \geq 1$.

Assume w is stored in array $A[1..n]$

```
IsInL*(A[1..n]):  
  If (n = 0) Output YES  
  If (IsInL(A[1..n]))  
    Output YES  
  Else  
    For (i = 1 to n - 1) do  
      If IsInL*(A[1..i]) and IsInL(A[i + 1..n])  
        Output YES  
  
  Output NO
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Question: How many distinct sub-problems does $\text{IsInL}^*(A[1..n])$ generate? $O(n)$

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Example

Consider string *samiam*

Naming subproblems and recursive equation

After seeing that number of subproblems is $O(n)$ we **name** them to help us understand the structure better.

ISL^{*}(i): a boolean which is 1 if $A[1..i]$ is in L^* , 0 otherwise

Base case: **ISL^{*}(0) = 1** interpreting $A[1..0]$ as ϵ

Recursive relation:

- **ISL^{*}(i) = 1** if
 $\exists j, 0 \leq j < i$ s.t. **ISL^{*}(j)** and **IsInL(A[j + 1..i])**
- **ISL^{*}(i) = 0** otherwise

Output: **ISL^{*}(n)**

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Removing recursion to obtain iterative algorithm

Typically, after finding a dynamic programming recursion, we often convert the recursive algorithm into an iterative algorithm via explicit memoization and bottom up computation.

Why? Mainly for further optimization of running time and space.

How?

- First, allocate a data structure (usually an array or a multi-dimensional array that can hold values for each of the subproblems)
- Figure out a way to order the computation of the sub-problems starting from the base case.

Caveat: Dynamic programming is not about filling tables. It is about finding a smart recursion. First, find the correct recursion.

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Iterative Algorithm

```
IsStringInLstar-Iterative( $A[1..n]$ ):  
  boolean  $ISL^*[0..(n + 1)]$   
   $ISL^*[0] = TRUE$   
  for  $i = 1$  to  $n$  do  
    for  $j = 0$  to  $i - 1$  do  
      if ( $ISL^*[j]$  and  $IsInL(A[j + 1..i])$ )  
         $ISL^*[i] = TRUE$   
        break  
  
  if ( $ISL^*[n] = 1$ ) Output YES  
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- Running time: $O(n^2)$ (assuming call to $IsInL$ is $O(1)$ time)
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(for now)