11.4.2

Quick select

QuickSelect

Divide and Conquer Approach

1. Pick a pivot element a from A
2. Partition A based on a.
   $A_{\mathrm{less}} = \{x \in A \mid x \leq a\}$ and $A_{\mathrm{greater}} = \{x \in A \mid x > a\}$
3. $|A_{\mathrm{less}}| = j$: return a
4. $|A_{\mathrm{less}}| > j$: recursively find jth smallest element in A_{less}
5. $|A_{\mathrm{less}}| < j$: recursively find kth smallest element in A_{greater} where $k = j - |A_{\mathrm{less}}|$.

Example

16	14	34	20	12	5	3	19	11

Time Analysis

1. Partitioning step: $O(n)$ time to scan A
2. How do we choose pivot? Recursive running time?

Suppose we always choose pivot to be $A[1]$.

Say A is sorted in increasing order and $j = n$.
Exercise: show that algorithm takes $\Omega(n^2)$ time

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A Better Pivot

Suppose pivot is the ℓth smallest element where $n/4 \leq \ell \leq 3n/4$.
That is pivot is approximately in the middle of A
Then $n/4 \leq |A_{\text{less}}| \leq 3n/4$ and $n/4 \leq |A_{\text{greater}}| \leq 3n/4$. If we apply recursion,

$$T(n) \leq T(3n/4) + O(n)$$

Implies $T(n) = O(n)$!

How do we find such a pivot? Randomly? In fact works!
Analysis a little bit later.

Can we choose pivot deterministically?

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THE END

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(for now)