1   A **basic arithmetic expression** is composed of characters from the set $\{1, +, \times\}$ and parentheses. Almost every integer can be represented by more than one basic arithmetic expression. For example, all of the following basic arithmetic expression represent the integer 14:

$$1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1$$
$$((1 + 1) \times (1 + 1 + 1 + 1 + 1)) + ((1 + 1) \times (1 + 1))$$
$$(1 + 1) \times (1 + 1 + 1 + 1 + 1 + 1 + 1)$$
$$(1 + 1) \times (((1 + 1 + 1) \times (1 + 1)) + 1)$$

Describe and analyze an algorithm to compute, given an integer n as input, the minimum number of 1's in a basic arithmetic expression whose value is equal to n. The number of parentheses doesn't matter, just the number of 1's. For example, when $n = 14$, your algorithm should return 8, for the final expression above. The running time of your algorithm should be bounded by a small polynomial function of n.

To think about later:

2   Suppose you are given a sequence of integers separated by $+$ and $-$ signs; for example:

$$1 + 3 - 2 - 5 + 1 - 6 + 7$$

You can change the value of this expression by adding parentheses in different places. For example:

$$1 + 3 - 2 - 5 + 1 - 6 + 7 = -1$$
$$(1 + 3 - (2 - 5)) + (1 - 6) + 7 = 9$$
$$(1 + (3 - 2)) - (5 + 1) - (6 + 7) = -17$$

Describe and analyze an algorithm to compute, given a list of integers separated by $+$ and $-$ signs, the maximum possible value the expression can take by adding parentheses. Parentheses must be used only to group additions and subtractions; in particular, do not use them to create implicit multiplication as in $1 + 3(-2)(-5) + 1 - 6 + 7 = 33$.