Algorithms for Regular and CFG Languages

In the following, let $M = (Q, \Sigma, \delta, s, A)$ be a DFA with n states, over an alphabet Σ of constant size.

- **1** Describe an algorithm for deciding if $L(M) = \emptyset$.
- **2** Describe an algorithm for deciding if $L(M) = \Sigma^*$.
- **3** Describe an algorithm for deciding if L(M) is finite.
- 4 Given two DFAs M, M' decide if $L(M) \subseteq L(M')$.
- **5** Given two DFAs M, M' decide if $L(M) = \overline{L(M')} = \Sigma^* \setminus L(M')$.
- **6** Given a CFG G = (V, T, P, S), decide if L(G) contains any string.
- **7** Two sets $q, q' \in Q$, are distinguishable, if there exists a string $w \in \Sigma^*$, such that $\delta(q, w) \in A$ and $\delta(q', w) \notin A$ (or vice versa). Show how to compute the set D_0 of all the pairs of states that are distinguishable with strings of length 0.
- 8 Let D_i be all the set of pairs of states of M that are distinguishable with strings of length at most i. Show how to compute D_{i+1} from D_i . (Think about i = 0 first, and then i = 1, etc.)
- 9 One can show that if $D_i = D_{i+1}$, then D_i is the set of all distinguishable pairs of states of M. Since $|D_i| \leq {n \choose 2}$, it follows that this happens after at most $O(n^2)$ iterations of the algorithm using the above steps. Let D^* be the set of pairs the first iteration this happens this is the set of all distinguishable pairs of states of M. Given M and D^* , show how to compute a minimal automata equivalent to M.