
For each of the following languages over the alphabet $\Sigma = \{0, 1\}$, either prove the language is regular (by giving an equivalent regular expression, DFA, or NFA) or prove that the language is not regular (using a fooling set argument). Exactly half of these languages are regular.

1 $\{0^n 10^n \mid n \geq 0\}$

2 $\{0^n 10^n w \mid n \geq 0 \text{ and } w \in \Sigma^*\}$

3 $\{w0^n10^n x \mid w \in \Sigma^* \text{ and } n \geq 0 \text{ and } x \in \Sigma^*\}$

4 Strings in which the number of 0s and the number of 1s differ by at most 2.

5 Strings such that *in every prefix*, the number of 0s and the number of 1s differ by at most 2.

6 Strings such that *in every substring*, the number of 0s and the number of 1s differ by at most 2.