This is a review of context-free grammars from the lecture on Tuesday; in each example, the grammar itself is on the left; the explanation for each non-terminal is on the right.

- Properly nested strings of parentheses.

$S \to \epsilon \mid S(S)$	properly nested parentheses

  Here is a different grammar for the same language:

$S \to \epsilon \mid (S) \mid SS$	properly nested parentheses

- $\{0^m 1^n \mid m \neq n\}$. This is the set of all binary strings composed of some number of 0s followed by a different number of 1s.

$S \to A \mid B$	$\{0^m 1^n \mid m \neq n\}$
$A \to 0A \mid 0C$	$\{0^m 1^n \mid m > n\}$
$B \to B1 \mid C1$	$\{0^m 1^n \mid m < n\}$
$C \to \epsilon \mid 0C1$	$\{0^m 1^n \mid m = n\}$

Give context-free grammars for each of the following languages. For each grammar, describe *in English* the language for each non-terminal, and in the examples above. As usual, we won't get to all of these in section.

1 $\{0^{2n} 1^n \mid n \geq 0\}$

2 $\{0^m 1^n \mid m \neq 2n\}$

(**Hint:** If $m \neq 2n$, then either $m < 2n$ or $m > 2n$. Extend the previous grammar, but pay attention to parity. This language contains the string 01.)

3 $\{0, 1\}^* \setminus \{0^{2n}1^n \mid n \geq 0\}$

(**Hint:** Extend the previous grammar. What is missing?)

Work on these later:

4 $\left\{w \in \{0, 1\}^* \;\middle|\; \#(0, w) = 2 \cdot \#(1, w)\right\}$ – Binary strings where the number of 0s is exactly twice the number of 1s.

5 $\{0, 1\}^* \setminus \{ww \mid w \in \{0, 1\}^*\}$.

[Anti-hint: The language $\{ww \mid w \in 0, 1^*\}$ *is* ***not*** *context-free. Thus, the complement of a context-free language is not necessarily context-free!]*