HW 4

Due on **Wednesday, September 25, 2024 at 9am CST**

CS/ECE 374A: Intro. Algorithms & Models of Computation, Fall 2024 Version: **1.0**

Submission instructions as in previous homeworks.

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7 (100 PTS.) Fool me once.

You are given a DFA $M = (Q, \Sigma, \delta, s, A)$ over the alphabet $\Sigma = \{0, 1\}$. The purpose of this question is to describe an efficient (i.e., polynomial time) algorithm to compute the minimal automata (in the number of stats) M' such that $L(M') = L(M)$.

7.A. (20 PTS.) Two states $q, q' \in Q$ are ***equivalent*** if

$$\forall w \in \Sigma^* \qquad \delta(q, w) \in A \iff \delta(q', w) \in A.$$

Describe how to check (efficiently [i.e., polynomial running time in the input size]) if L_q and $L_{q'}$ are the equal, where

$$L_q = \{w \in \Sigma^* \mid \delta(q, w) \in A\}$$

is the ***language of*** q. Using this, describe how to decide if q and q' are equivalent. (Hint: How do you decide if two automatas accept the same language?)

7.B. (50 PTS.) Given that q and q' are equivalent, in the given DFA $M = (Q, \Sigma, \delta, s, A)$, and $q \neq q'$, describe formally how to construct (efficiently) a new DFA M' with $n - 1$ states, such that $L(M') = L(M)$, where $n = |Q|$. ***Prove*** that your construction is correct.

(Hint: For the proof of correctness, you need to prove that the languages of the states of the new automata are the same as they were in the original automata. This probably should be done by induction on the length of the strings.)

7.C. (30 PTS.) Using the above, and only the above, describe how to construct a DFA M' with $L(M') = L(M)$, such that no two states in M' are equivalent. How many invocations of the algorithm of part B your algorithm requires in computing M'?

(The Myhill-Nerode theorem implies M' is the minimal automata for $L(M)$.)

8 (100 PTS.) Fool me twice.

Let $\Sigma = \{0, 1, 2\}$. Consider the language L of all strings $w \in \Sigma^*$ where $\#_0(w) = \#_1(w) + \#_2(w)$.

8.A. (30 PTS.) Prove that this language is CFG by providing a grammar G for it.

8.B. (70 PTS.) Prove that your grammar indeed yields the desired language (i.e., prove that $L(G) \subseteq L$ and $L \subseteq L(G)$).