HW 2

Due on **Wednesday, September 11, 2024 at 9am CST**

CS/ECE 374A: Intro. Algorithms & Models of Computation, Fall 2024 Version: **1.0**

Submission instructions as in previous homeworks.

3 (100 PTS.) Regular.

For each of the following languages, give a regular expression that accepts that language, and briefly argue why your expression is correct. Below, $\#_0(x)$ denotes the number of 0s in x.

You do not need to provide the shortest [or even short] regular expression that works – instead, try to provide a systematic solution explaining how you reached your answer. Please provide an explicit and full regular expression.

3.A. (25 PTS.) All strings in $\{0, 1\}^*$ that do not contain 0101010 as a subsequence.

3.B. (25 PTS.) All strings in $\{0, 1\}^*$ such that the symbols at even positions are alternating. For example: the string $0\underline{0}0\underline{1}1\underline{0}0\underline{1}1\underline{0}1$ is in the language because the underlined characters alternate between 0 and 1. While $1\underline{0}1\underline{1}1\underline{0}0\underline{0}1\underline{0}1$ is not in the language.

(Hint: Start with a regular expression for all strings that all their bits are alternating, and then extend it to the desired expression.)

3.C. (25 PTS.) All strings $x \in \{0, 1\}^*$, such that x does not begin with 010 and $\#_0(x)$ is even.

(Hint: First come up with a regular expression for all strings with even (and separately odd) number of 0s. Then create a regular expression for all the strings in the language starting with 1, etc.)

3.D. (25 PTS.) All strings in $\{0, 1\}^*$ that do not contain 010 as a substring.

(Hint: Generate a regular expression for all strings in this language that starts with a 0 and ends with a 0. Once you have this regular expression, getting the answer is shockingly easy.)

4 (100 PTS.) Divisible by something.

In the following, you need to explain (shortly) why your solution works (a formal proof is not necessary).

4.A. (50 PTS.) Let $\Sigma = \{0, 1\}$. For a string $w \in \Sigma^*$, let w_2 be the integer value if we interpret w as a number written in base 2. Thus, $1010_2 = 1 \cdot 2^3 + 1 \cdot 2^1 = 10$.

Describe *formally* a DFA that accepts the language L of all strings $w \in \Sigma^*$, such that $(w^R)_2$ is divisible by 13. For example, $001011 \in L$, since $(001011^R)_2 = 110100_2 = 13 \cdot 4$, which is divisible by 13, as is $10111011 \in L$. But $001 \notin L$, since $100_2 = 4$, which is not (yet) divisible by 13.

(Hint: Think about the DFA as giving you the input from right to left.)

4.B. (50 PTS.) A string $w \in \Sigma^*$ is a k-palindrome, for a prespecified integer $k > 1$, if k divides w_2, and k also divides $(w^R)_2$. Describe *formally* a DFA that accepts all strings $w \in \Sigma^*$ that are 13-palindrome.