HW 1

Due on **Wednesday, September 4, 2024 at 9am CST**

CS/ECE 374A: Intro. Algorithms & Models of Computation, Fall 2024 Version: **1.0**

- Guidelines for homework submissions are specified here:
  https://courses.grainger.illinois.edu/cs374al1/fa2024/info/hw_policies.html
  Read them **carefully** before submitting your homework.

Some important course policies

- **You may use any source at your disposal** – paper, electronic, or human – but you *must* cite *every* source that you use, and you *must* write everything yourself in your own words. See the academic integrity policies on the course web site for more details.
- **Avoid the Three Deadly Sins!** Any homework or exam solution that breaks any of the following rules will be given an *automatic zero*, unless the solution is otherwise perfect. Yes, we really mean it. We are not trying to be scary or petty (Honest!), but we do want to break a few common bad habits that seriously impede mastery of the course material.
  - Always give complete solutions, not just examples.
  - Always declare all your variables, in English. In particular, always describe the specific problem your algorithm is supposed to solve.
  - Short **complete** answers are better than longer answers. Unnecessarily long answers (which by definition are not perfect) would get zero (i.e., 0) points. Avoid empty expressions like "in fact", "as anybody, or their uncle, can see if they think about it...", etc.
  - Always give credit to outside sources! (Yes, we are no good with counting, and we repeat ourselves unnecessarily [which you should not do].)

See the course web site for more information.

If you have any questions about these policies,
please do not hesitate to ask in class, in office hours, or on EdStem.

Extra problems (one or more fully solved) are available in the https://courses.engr.illinois.edu/cs374/fa2022/hw/hw_01_extra.pdfHW 1 extra problems collection on the class webpage: https://courses.engr.illinois.edu/cs374/fa2022. It is recommended that you look on these extra problems before doing the homework, since it would help you with doing the homeworks. These are also good practice problems for the midterms and final.

You also need to do (individually) also a question on PrairieLearn, which is due on **Tuesday, September 3, 2024 at 10am CST**.

1 (100 PTS.) Moo = $\overline{\text{ooM}}$?

Let $\Sigma = \{0, 1\}^*$. A string $w \in \Sigma^*$ is a ***reversi***, if $\overline{w^R} = w$. That is, reversing w and flipping all its bits (i.e., characters) results in the same string. So 01 is a reversi (since $01^R = 10$, and thus $\overline{01^R} = \overline{10} = 01$. Similarly, 0101 is also a reversi, but 10101 is not a reversi. Let $\mathcal{R}$ be the set of all reversi strings in Σ^*.

Consider the language $L \subseteq \Sigma^*$, where:

(i) $\varepsilon \in L$.

(ii) For all $x \in L$, the strings $0x1$ and $1x0$ are also in L.

Specifically, the language L contains only strings that can be constructed by the above rules.

(In the following, you can use [without proof] that for any strings $x, y \in \Sigma^*$, we have $(xy)^R = y^R \bullet x^R = y^R x^R$, and $\overline{xy} = \overline{x} \bullet \overline{y} = \overline{x}\,\overline{y}$.)

1.A. (50 PTS.) Prove (by induction) that $L \subseteq \mathcal{R}$.

1.B. (50 PTS.) Prove (by induction) that $\mathcal{R} \subseteq L$ (thus $L = \mathcal{R}$ – that is, L is the language of all reversi strings).

2 (100 PTS.) Ohh, recurrence.

Consider the following recurrence:

$$T(n) = \begin{cases} 26\,T(\lfloor n/3 \rfloor) + 8\,T(\lfloor n/6 \rfloor) + 2024 & \text{n} > 27 \\ 225 & 1 \le n \le 27. \end{cases}$$

Prove that $T(n) = O(n^3)$ by induction.

[Hint: you may need to use an induction hypothesis of the form $T(n) \le cn^3 - c'$ for some constants c and c' to be determined.]