CS/ECE 374: Algorithms & Models of Computation, Fall 2017
Midterm 1 October 2: 7–9pm, 2017

A	B	C	D	E	F	G	H	J	K	?
9am	10am	11am	noon	1pm	1pm	2pm	2pm	3pm	3pm	Waiting
Rucha	Rucha	Srihita	Shant	Abhishek	Xilin	Shalan	Phillip	Vishal	Phillip	list
SC 1404	SC 1404	SC 1404	DCL 1320	DCL 1320	DCL 1320	ECE 1002	ECE 1002	ECE 1002	ECE 1002	ECE 1002

Name:	
NetID:	

#	1	2	3	4	5	Total
Score						
IDK Score						
Max	20	20	20	20	20	100
Grader						

- ***Don't panic!***
- Please print your name and NetID **in each page** in the appropriate fields, and circle your discussion section in the boxes above. We will return your exam at the indicated section.
- If you brought anything except your writing implements, your double-sided **handwritten** (in the original) 8½" × 11" cheat sheet, and your university ID, please put it away for the duration of the exam. In particular, you please turn off and put away *all* medically unnecessary electronic devices.
  - Submit your cheat sheet together with your exam. An exam without your cheat sheet attached to it will not be graded.
  - If you are NOT using a cheat sheet, please indicate so in large friendly letters on this page.
- Please ask for clarification if any question is unclear.
- **This exam lasts 120 minutes.** The clock started when you got the questions.
- If you run out of space for an answer, feel free to use the blank pages at the back of this booklet, but please tell us where to look.
- As usual, answering any (sub)problem with I don't know (and nothing else) is worth 25% partial credit. Correct, complete, but sub-optimal solutions are *always* worth more than 25%. A blank answer is not the same as I don't know.
- Total IDK points for the whole exam would not exceed 10.
- **Beware the Three Deadly Sins.** Give complete solutions, not examples. Declare all your variables. If you don't know the answer admit it and use IDK.
- **Style counts.** Please use the backs of the pages or the blank pages at the end for scratch work, so that your actual answers are clear.
- Please return ***all*** paper with your answer booklet: your cheat sheet, and all scratch paper. We will return everything with your graded exam.
- ***Good luck!***

NETID: NAME:

1 For each statement below, check "True" if the statement is ***always*** true and "False" otherwise. Each correct answer is worth 2 points; each incorrect answer is worth 0 points; and flipping a coin is (on average) worth 1 point. **There is no IDK for this question**.

1.A.	For all languages L, if L is not regular, then L does not have a finite fooling set.	**False**: ☐	**True**: ☐
1.B.	For all context-free languages L and L', the language $L \cup L'$ is also context-free.	**False**: ☐	**True**: ☐
1.C.	$\{0^i 1^j 0^k 1^\ell \mid i \geq j \geq k \geq \ell \geq 0\}$ is not regular.	**False**: ☐	**True**: ☐
1.D.	Let $M = (\Sigma, Q, s, A, \delta)$ and $M' = (\Sigma, Q, s, Q \setminus A, \delta)$ be arbitrary NFAs with identical alphabets, states, starting states, and transition functions, but with complementary accepting states. Then $L(M) \setminus L(M') = L(M)$.	**False**: ☐	**True**: ☐
1.E.	For all languages $L, L' \subset \Sigma^*$, if L and L' are recognized by DFAs M and M', respectively, then $L' \setminus L$ can be represented by a regular expression.	**False**: ☐	**True**: ☐
1.F.	If $L_1, L_2, \ldots$ are all context free languages, then $\bigcup_{i=0}^{\infty} L_i$ is context free.	**False**: ☐	**True**: ☐
1.G.	Let L be a regular language over alphabet Σ, and consider the language $$L' = \{x\alpha y \mid x, y \in \Sigma^*, \alpha \in \Sigma, \text{ and } xy \in L\}.$$ The language L' is not regular.	**False**: ☐	**True**: ☐
1.H.	The strings 010 and 101 are distinguishable by the language $$L = \{x \in \Sigma^* \mid \text{the number of 1s in } x \text{ is even}\}.$$	**False**: ☐	**True**: ☐
1.I.	If a language $L \subseteq \{0\}^*$ then the language L^* is regular.	**False**: ☐	**True**: ☐
1.J.	Consider the logical statement "If pigs can fly then $2 + 2 = 5$." This expression is:	**False**: ☐	**True**: ☐

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2 For each of the following languages over the alphabet $\Sigma = \{0, 1\}$, either ***prove*** that the language is regular or ***prove*** that the language is not regular. ***Exactly one of these two languages is regular.*** [This is a tricky question.]

2.A. $L = \left\{ 1^n w 0^{m-n} \;\middle|\; w \in \Sigma^+ \text{ and } m \geq n > 0 \right\}$.

2.B. $L = \left\{ x 0^n x^R \;\middle|\; x \in \Sigma^+ \text{ and } n > 0 \right\}$, where x^R is the *reverse* string of x.

3 For any language L, let $\textsc{ps}(L) = \{xz \mid xyz \in L \text{ for some } x, y, z \in \Sigma^*\}$ be the language containing all strings in L after one is allowed to delete some continuous middle portion. For example, if $L = \{000, 10101\}$, then we have the string $10\cancel{10}1 = 101 \in \textsc{ps}(L)$. In this case, we have

$$\textsc{ps}(L) = \left\{ \ \varepsilon, 0, 00, 000, 10101, 0101, 1101, 1001, 1011, 1010, 101, 01, 11, 10, 1 \ \right\}.$$

Prove that for any regular language L, the language $\textsc{ps}(L)$ is also regular. Your proof should be self contained, and not use any languages seen in class/lectures (suggestion: first describe the necessary construction, and then prove the correctness of the construction).

4 In the following, you do ***not*** need to prove that your answers are correct.

4.A. Provide a DFA and a regular expression for the following language: The set of all strings in $\{a, b\}^*$ that do not contain the substring $aaaaa$.

4.B. Provide a regular expression for the following language: The set of all strings in $\{a, b\}^*$ that contain both ab and aa as substrings.

5 A CFG G is *idiotic* if all production rules in G are of the form $A \rightarrow wB$ or $A \rightarrow \varepsilon$, where $w \in \Sigma^+$ and A and B are variables (i.e.., non-terminals) of G. Here $\Sigma = \{0\}$. For a given idiotic CFG grammar G, provide a construction that shows that the language of $L(G)$ is regular. Provide a convincing argument why this is true.

[No need to provide a formal proof here - it is too tedious for an exam.]

[If you do not know the answer - just use IDK – no need to waste your and our time.]