CS/ECE 374A: Intro. Algorithms & Models of Computation, Fall 2024

Midterm 1 — **Monday, September 30, 2024: 7-9pm**

Location: CIF 0025

Name		⇐ Please PRINT
NetID		⇐ Please PRINT

- Please print your name and NetID. Anonymous submissions would not be graded.
- There are four questions – you should answer all of them.
- If you brought anything except your writing implements, your double-sided **handwritten** (in the original, by yourself) $8\frac{1}{2}" \times 11"$ cheat sheet, and your university ID, please put it away for the duration of the exam. In particular, please turn off and put away *all* medically unnecessary electronic devices.
  - Submit your cheat sheet together with your exam. We will not return or scan the cheat sheets, so photocopy them before the exam if you want a copy.
  - If you are NOT using a cheat sheet, please indicate so in large friendly letters on this page.
- Please read all the questions before starting to answer them. Please ask for clarification if any question is unclear.
- **This exam lasts 120 minutes.** The clock started when you got the questions.
- If you run out of space for an answer, feel free to use the blank pages at the back of this booklet, but please tell us where to look.
- Give complete solutions, not examples. Declare all your variables. If you don't know the answer admit it and move to the next question. We will happily give 0 points for nonsense answers.
- **Style counts.** Please use the backs of the pages or the blank pages at the end for scratch work, so that your actual answers are clear.
- Please return ***all*** paper with your answer booklet: your exam, your cheat sheet, and all scratch paper.
- Do not write outside the framed area, do not remove the staple, and do not remove the last page of the booklet.
- ***Good luck!***

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1 (20 PTS.) For each statement below, check "True" if the statement is ***always*** true and "False" otherwise. Each correct answer is worth two points; each incorrect answer is worth nothing.

1.A. If $L_1 \subseteq L_2 \subseteq L_3 \subseteq \cdots$ are all context free languages, then the language $\bigcap_i^\infty L_i$ is context-free. **False:** ☐ **True:** ☐

1.B. If all the fooling sets for a given language L are finite, then L is regular. **False:** ☐ **True:** ☐

1.C. Let $\Sigma = \{0, 1, 2\}$, and let $L \subseteq \Sigma^*$ be a language that is not regular. For any string $w \in \Sigma^*$, there is an infinite fooling set F for L such that $w \in F$. **False:** ☐ **True:** ☐

1.D. Let $L_1, L_2, \ldots, L_{201} \subseteq \{0,1\}^*$ be regular languages. Let L be the ***majority language*** of these languages – that is, $w \in L \iff w$ appears in at least 101 of the languages $L_1, \ldots, L_{201}$. The language L is regular. **False:** ☐ **True:** ☐

1.E. For any context-free language L and a regular language L', the language $L' \setminus L$ is context-free. **False:** ☐ **True:** ☐

1.F. For all languages $L_1, L_2 \subseteq \{0,1\}^*$, if L_1 and L_2 are accepted by DFAs M_1 and M_2, respectively, then the language

$$L' = \left\{ w \in \Sigma^* \;\middle|\; w \in L_1 \text{ and } \overline{w}^R \in L_2 \right\}$$

can be represented by a regular expression. Here $\overline{w}$ is the result of bitwise negating each of the bits. E.g., $\overline{01} = 10$. **False:** ☐ **True:** ☐

1.G. Let $M = (\Sigma, Q, s, A, \delta)$ and $M' = (\Sigma, Q, s, A', \delta)$ be arbitrary NFAs with identical alphabets, states, starting states, and transition functions, but with different accepting states, such that A is a proper subset of A'. Then it must be that $L(M) \subset L(M')$ and $L(M) \neq L(M')$. **False:** ☐ **True:** ☐

1.H. The language $\left\{ 0^i 1^j 0^k 1^\ell \;\middle|\; (i \cdot j) \bmod 7 = (k \cdot \ell) \bmod 7 \right\}$ is regular. **False:** ☐ **True:** ☐

1.I. Let L be a regular language over alphabet $\Sigma = \{0, 1\}$, and consider the language

$$L' = \{x1x \mid x \in \Sigma^* \text{ and } xx \in L\}.$$

The language L' is regular. **False:** ☐ **True:** ☐

1.J. Consider a language $L \subseteq \{0\}^*$. If L^* is regular then L is regular. **False:** ☐ **True:** ☐

2 (25 PTS.) Provide a regular expression for each of the following languages (and explanation why the expression is correct):

2.A. (5 PTS.) (Homework.) All strings in $\{0, 1\}^*$ that do not contain 01010 as a subsequence.

2.B. (10 PTS.) (Discussion section problem.) All binary strings such that in every prefix, the number of 0s and the number of 1s differ by at most one.

2.C. (10 PTS.) (Fodder.) All binary strings in which every run of consecutive 0s has even length and every run of consecutive 1s has odd length, and the first character in the string is 1.

3 (25 PTS.) CONTEXT FREE LANGUAGES.

In the following, provide a detailed (but short) explanations for your answers (proof is not required).

3.A. (10 PTS.) Describe a context-free grammar (CFG) for the language L of all strings $w \in \Sigma^*$, for $\Sigma = \{0,1\}^*$, such that one has to negate at most $k = 2$ characters in w to get a palindrome. Thus $0101 \in L$, since $01\overline{0}\,\overline{1} = 0110$ is a palindrome, but $010101 \notin L$. (Hint: First handle the cases $k = 0$ and $k = 1$.)

3.B. (15 PTS.) Let $\Sigma = \{0,1\}$, and let $U = \left\{0^i 1^i \;\middle|\; i \geq 0\right\}$. Describe a CFG for the complement language $\overline{U} = \Sigma^* \setminus U$.

4 (30 PTS.)

4.A. (15 PTS.) Let $\mathrm{swap}(w)$ be the set of all strings formed by selecting any number of disjoint pairs of consecutive bits in w, and then swapping the bits within each pair. For example, $\mathrm{swap}(01) = \{01, 10\}$, and

$$\mathrm{swap}(0101) = \{0101, \underline{10}01, 0\underline{01}1, 01\underline{10}, \underline{10}\,\underline{10}\}$$

For all regular language L, the language $L' = \mathrm{swap}(L) = \bigcup_{w \in L} \mathrm{swap}(w)$ is regular.
Given a DFA $M = (Q, \Sigma, \delta, s, A)$ for L, with $\Sigma = \{0, 1\}$, describe how to construct an NFA N for the language L'. Explain in detail why your construction is correct.
(Part (B) is on the other side of the page.)

(See part (B) on the other side.)

4.B. (15 PTS.) Let $\text{swap}(x, i)$ be the set of all strings that one can generate from x by applying swap() to it (at most) i times. Let $\text{SWAP}(x) = \cup_{i=1}^{\infty} \text{swap}(w, i)$. For example $\text{SWAP}(0101) = \{0011, 0101, 1001, 0110, 1010, 1100\}$.

As a further example, $0101 \rightsquigarrow \underline{10}\,\underline{10} \rightsquigarrow 1\underline{10}0 \in \text{SWAP}(0101)$.

Provide a regular language L, such that the language $L' = \text{SWAP}(L) = \bigcup_{w \in L} \text{SWAP}(w)$ is not regular. ***Prove*** that L' is not regular for your choice of L.

(scratch paper)

(scratch paper)