Final: Monday, December 18, 8-11am, 2017

A	B	C	D	E	F	G	H	J	K
9am	10am	11am	noon	1pm	1pm	2pm	2pm	3pm	3pm
Rucha	Rucha	Srihita	Shant	Abhishek	Xilin	Shalan	Phillip	Vishal	Phillip
101 Armory	101 Armory	101 Armory	151 Loomis	151 Loomis	151 Loomis	ECE 1002	ECE 1002	ECE 1002	ECE 1002

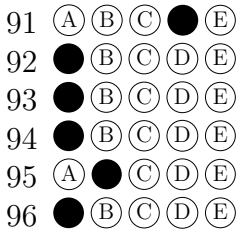
Name:	
NetID:	
Name on Gradescope:	

- ***Don't panic!***
- If you brought anything except your writing implements, your double-sided **handwritten** (in the original) 8½" × 11" cheat sheet, and your university ID, please put it away for the duration of the exam. In particular, please turn off and put away *all* medically unnecessary electronic devices.
  - Submit your cheat sheet together with your exam. An exam without your cheat sheet attached to it will not be graded.
  - If you are NOT using a cheat sheet, please indicate so in large friendly letters on this page.
- **Best answer.** Choose best possible choice if multiple options seems correct to you – for algorithms, faster is always better.
- Please ask for clarification if any question is unclear.
- **This exam lasts 170 minutes.**
- Fill your answers in the Scantron form using a pencil. We also recommend you circle/mark your answer in the exam booklet.
- Please return ***all*** paper with your answer booklet: your cheat sheet, and all scratch paper. We will **not** return the cheat sheet.
- Do not fill more than one answer on the Scantron form - such answers would not be graded. Also, fill your answer once you are sure of your answer – erasing an answer might make the form unscannable.
- ***Good luck!***

Before doing the exam...

- Fill your name and netid in the back of the Scantron form, and also on the top of this page.
- **Fill in the pattern shown on the right in the Scantron form.**

  This encodes which version of the exam you are taking, so that we can grade it.



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1. (1 point) All problems in P are solvable in exponential time. This statement is

(A) False.
(B) True.

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2. (3 points) You are given a directed graph G with n vertices and m edges. Consider the problem of deciding if there is a closed walk (the walk is allowed to repeat both vertices and edges) that visits all the vertices of G.

(A) This problem is NP-Complete.
(B) This problem can be solved in $O(n + m)$ time.
(C) This problem is NP-Hard.
(D) All of the other answers are correct.
(E) This problem can be solved in $O(nm)$ time, and no faster algorithm is possible.

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3. (3 points) Let G be a directed graph with weights on the edges (the weights can be positive or negative). The graph G has n vertices and m edges. Computing the shortest **simple** path between two vertices in G can be done in:

(A) This is not defined if there are negative cycles in the graph. As such, it can not be computed.
(B) This can be solved in $O(nm)$ time using Bellman-Ford.
(C) This can be solved in $O(n \log n + m)$ time using Dijkstra.
(D) None of the above.
(E) This is NP-Hard.

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4. (3 points) Let $\mathcal{PC}$ be the class of all decision problems, for which there is a polynomial time certifier that works in polynomial time, and furthermore, for an input of length n, if it is a YES instance, then there is a certificate that is a binary string of length $n^{O(1)}$. We have that:

(A) All the problems in $\mathcal{PC}$ can be solved in polynomial time.
(B) $\mathcal{PC}$ contains some NP-Complete problems, but not all of them.
(C) NP $\subsetneq \mathcal{PC}$.
(D) NP $= \mathcal{PC}$.
(E) None of the other answers is correct.

5. (2 points) You are given a directed graph G with n vertices and m edges, a pair of vertices u, v, and a number t. Deciding if there is a closed walk, with at most t edges, that includes u and v can be done in

(A) $O(n \log n + m)$ time.
(B) only exponential time since this problem is NP-Complete.
(C) $O(n + m)$ time.
(D) $O(nm)$ time using Bellman-Ford.
(E) None of the other answers is correct.

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6. (2 points) Consider the following decision problem: Given a directed graph G, and two vertices u, v in G, is there a path from u to v in G?
This problem has a polynomial length certificate and polynomial time certifier. This claim is

(A) True.
(B) False.

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7. (3 points) For the following recurrence (evaluated from top to bottom in this order):

$$f(i,j) = \begin{cases} 1 & j > i \text{ or } j < 0 \text{ or } i < 0 \\ 2 & j = 0 \text{ or } j = i \\ f(i-1, j-1)\log(1 + f(i-1, j)) & \text{otherwise.} \end{cases}$$

Assume that every arithmetic operation takes constant time (even if the numbers involved are large). Computing $f(n, \lfloor n/2 \rfloor)$ can be done in (faster is better):

(A) $O(2^n)$.
(B) $O(n \log n)$ time.
(C) $O(n^2)$ time, using dynamic programming.
(D) $O(\log n)$.
(E) $O(n)$ time, by recursion.

8. (3 points) For a text file T, let $\langle T \rangle$ denote the string that is the content of T. Consider the language

$$L = \{\langle T \rangle \mid T \text{ is a java program that stops on some input}\}.$$

This language is

(A) Decidable.
(B) Context-free.
(C) Regular.
(D) None of the other answers.
(E) Undecidable.

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9. (3 points) You are given a directed graph G with n vertices, m edges, and positive weights on the vertices (but not on the edges). In addition, you are given two vertices u and v, and a number t. The *weight* of a path π is the total weight of the vertices of π. Consider the problem of computing a (simple) path σ connecting a vertex u to a vertex v, such that the weight of σ is at least t. This problem is

(A) Solvable in $O(n + m)$ time.
(B) Solvable in $O(n \log n + m)$ time.
(C) NP-Hard.
(D) Undecidable.
(E) Polynomially equivalent to Eulerian cycle.

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10. (3 points) You are given a graph G with n vertices and m edges, and with weights on the edges. In addition, you are given the MST tree T.
Next, you are informed that the price of some edge e in the graph G had decreased from its current cost, to a new cost α. Deciding if T is still the MST of the graph with the updated weights can be done in (faster is better):

(A) $O(\log n)$ time, after preprocessing the graph in $O(n)$ time.
(B) $O(n \log n + m)$ time.
(C) None of the other answers.
(D) $O(nm)$ time algorithm, and no faster algorithm is possible.
(E) $O(n)$ time.

11. (3 points) Consider a CNF formula F with all clauses being of size 2, except for 10 clauses that are of size at most 7 (i.e., these clauses are made out of up to seven literals). Consider the problem of deciding if such a formula is satisfiable. We have:

(A) Can be solved in linear time.
(B) None of the other answers are correct.
(C) NP-Hard.
(D) Can not be solved in linear time, but can be done in polynomial time.

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12. (3 points) Let $P_1, \ldots, P_{k+1}$ be $k+1$ decision problems. Consider a sequence of k polynomial reductions $R_1, \ldots, R_k$, where R_i works in quadratic time in its input size, and is a reduction from P_i to P_{i+1}. As such, there is a reduction from P_1 to P_{k+1} and its running time is

(A) $O\left(n^{2k}\right)$
(B) $O(k^2 n^2)$
(C) $O\left(2^k n^2\right)$
(D) $O\left(n^{2^k}\right)$
(E) $O(kn^2)$

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13. (3 points) For the language $L = \{a^n b^n \mid n \geq 0\}$, we have

(A) $F = \left\{a^i b^j \;\middle|\; i < j\right\}$ is a fooling set for L.
(B) All of the sets suggested are fooling sets.
(C) $F = \{a^i \mid i \geq 0\}$ is a fooling set for L.
(D) $F = \left\{a^i b^i \;\middle|\; i \geq 0\right\}$ is a fooling set for L.
(E) None of the sets suggested are fooling sets.

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14. (3 points) You are given a directed graph G with n vertices and m edges. Deciding if this graph has a simple cycle visiting at least 1000 edges is

(A) Doable in $O(n+m)$ time.
(B) Doable in polynomial time (but not linear time).
(C) NP-Complete by a reduction from this problem to Hamiltonian path/cycle.
(D) None of other answers are correct.
(E) NP-Complete by a reduction from Hamiltonian path/cycle to this problem.

15. (2 points) Consider a Turing machine (i.e., program) M that accepts an input $w \in \Sigma^*$ if and only if there is a CFG G such that $w \in L(G)$. Then the language of $L(M)$ is

(A) context-free.
(B) Σ^*.
(C) undecidable.
(D) finite.
(E) not well defined.

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16. (2 points) You are given an unsorted set Y of m numbers. Deciding if there are two numbers x and y in Y such that $xy = 1$ can be solved in (faster is better):

(A) $O(m^2)$ time.
(B) $O(m^2 \log m)$ time.
(C) $O(m)$ time.
(D) $O(m \log m)$ time.
(E) $O(m^{3/2})$ time.

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17. (3 points) Consider the problem of checking if a graph can be colored by four colors (i.e., no adjacent pair of vertices have the same color). It can be solved in

(A) Polynomial time.
(B) None of the other answers are correct.
(C) Maybe polynomial time – we do not know. Currently fastest algorithm known takes exponential time.
(D) It is NP-Complete, so it can not be solved efficiently.

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18. (3 points) Consider the recurrence $f(n) = f\Big(\lfloor (2/3)n \rfloor\Big) + f\Big(\lfloor (1/3)n \rfloor\Big) + O(n)$, where $f(n) = O(1)$ if $n < 10$. The solution to this recurrence is

(A) $O(n)$.
(B) None of the above.
(C) $O(n \log n)$.
(D) $O(1)$.
(E) $O(n^2)$.

19. (3 points) The number of context-free languages, over the alphabet $\{0, 1\}$, is

(A) countable.
(B) $\aleph_2$.
(C) None of the other answers are correct.
(D) undecidable.
(E) uncountable.

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20. (2 points) Consider the language

$$L = \left\{ 1^i \;\middle|\; i \geq 0, i \text{ is an integer, and } i \text{ is } \underline{\text{not}} \text{ divisible by } p_1, p_2, \ldots, p_{100} \right\},$$

where p_j is the jth smallest prime number, for $j = 1, \ldots, 100$ (i.e., $p_1 = 2, p_3 = 3, \ldots, p_{100} = 541$). This language is

(A) Decidable.
(B) Finite.
(C) Regular.
(D) Undecidable.
(E) Context-free.

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21. (3 points) Given an undirected graph G with n vertices and m edges, and a number k, deciding if G has a spanning tree with maximum degree k is

(A) Can be done in $O(n + m)$ time.
(B) Can be done in polynomial time.
(C) Can be done in $O(n \log n + m)$ time, and there is no faster algorithm.
(D) NP-Complete.
(E) Can be done in $O((n + m) \log n)$ time, and there is no faster algorithm.

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22. (3 points) Give a CNF formula F with n variables, and m clauses, where every clause has exactly two literals (reminder: a literal is either a variable or its negation). Then, one can compute a satisfying assignment to F in:

(A) $O(n^2 + m^2)$ time.
(B) This is Satisfiability and it can not be solved in polynomial time unless $P = NP$.
(C) $O(n \log n + m)$ time.
(D) $O(n + m)$ time.
(E) $O(2^n - 2^m)$ time.

23. (2 points) You are given a set $\mathcal{I} = \{I_1, I_2, \ldots, I_n\}$ of n weighted intervals on the real line. Consider the problem of computing a value $x \in \mathbb{R}$, that maximizes the total weight of the intervals of $\mathcal{I}$ containing x. This problem:

(A) Can be done in polynomial time.
(B) Can be done in linear time.
(C) NP-Hard.
(D) Undecidable.
(E) NP-Complete.

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24. (3 points) You are given a DFA D with n states, and with the input alphabet being $\Sigma = \{0, 1\}$. Given a binary string $w \in \Sigma^*$ of length m, one can simulate D on a regular computer and decide if D accepts w. Which of the following is correct?

(A) This can be done in $O(2^n m)$ time, and no faster algorithm is possible.
(B) None of the other answers is correct.
(C) This can done in $O(n + m)$ time.
(D) This problem can not be done in polynomial time, because it is undecidable.
(E) This can be done in $O(n^m)$ time, and no faster algorithm is possible.

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25. (3 points) If a problem is NP-Hard, then it can also be undecidable. This statement is

(A) True if $P = NP$.
(B) False if $P = NP$.
(C) False.
(D) True.
(E) None of the other answers.

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26. (3 points) Let B be the problem of deciding if the shortest path in a graph between two given vertices is smaller than some parameter k (where the weights on the edges of the graph are positive).
Let C be the problem of deciding if a given instance of 2SAT formula is satisfiable. Pick the correct answer out of the following:

(A) There is a polynomial time reduction from C to B, but only if $P \neq NP$..
(B) None of the other answers is correct.
(C) There is no relation between the two problems, and no reduction is possible.
(D) There is a polynomial time reduction from B to C, but only if $P = NP$.
(E) There is a polynomial time reduction from B to C.

27. (3 points) Given an array $B[1 \,..\, n]$ with n real numbers (B is not sorted), consider the problem computing and printing out the smallest $\lfloor \log^3 n \rfloor$ numbers in B that are larger than the median number in B – the numbers should be output in sorted order. This can be done in

(A) $O(n)$ time, and no faster algorithm is possible.
(B) $O(\log^3 n)$ time, and no faster algorithm is possible.
(C) $O(n \log n)$ time, and no faster algorithm is possible.
(D) $O(\log^4 n)$ time, and no faster algorithm is possible.

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28. (3 points) Given two DFAs N_1 and N_2 with n_1 and n_2 states, respectively. Then there is a DFA M that accepts the language $L(N_1) \oplus L(N_2)$, where $A \oplus B = (A \setminus B) \cup (B \setminus A)$.

(A) True, and the number of states of M is at most $O(n_1 + n_2)$.
(B) True, and the number of states of M is at most $n_1 n_2$.
(C) False.
(D) None of the other answers is correct.
(E) True, and the number of states of M is at most $2^{n_1} 2^{n_2}$.

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29. (3 points) Given an undirected graph G, with n vertices and m edges, consider the decision problem of determining if the vertices of G can be colored (legally) by 2 colors (i.e., no adjacent pair of vertices have the same color). This problem is:

(A) Can be solved in polynomial time.
(B) Solvable in $O(n + m)$ time.
(C) Undecidable.
(D) As hard as the independent set problem.
(E) NP-Complete.

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30. (3 points) Given a graph G, and vertices u and v, these two vertices are ***robustly connected***, if for all sets X of at most 20 edges in G, we have that u and v are connected in $\mathsf{G} - X$, where $\mathsf{G} - X = (\mathsf{V}(\mathsf{G}), \mathsf{E}(\mathsf{G}) \setminus X)$ – the graph resulting from removing the edges of X from G. Consider the decision problem of deciding if u and v are robustly connected. This problem:

(A) Can be solved in polynomial time.
(B) NP-Hard.
(C) Can be solved in $O(n \log n + m)$ time.

31. (2 points) Consider an NFA N with m states defined over $\{0,1\}^*$. There is an equivalent regular expression r (i.e., $L(r) = L(N)$), such that

(A) r is of length at most $O(m \log m)$.
(B) r is of length at most $f(m)$, where f is some function that is not specified in the other answers.
(C) r is of length at most $O(m)$.
(D) none of other answers are correct.

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32. (2 points) For a word $w = w_1 w_2 \ldots w_{2m}$, with $w_i \in \{0,1\}^*$, let $w^{\text{ODD}} = w_1 w_3 w_5 \ldots w_{2m-1}$. If a language L is a regular language, then the language $L^{\text{ODD}} = \left\{ w^{\text{ODD}} \;\middle|\; w \in L \right\}$ is regular.

(A) True.
(B) False.

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33. (5 points) A string $s \in \{a,b\}^*$ is γ-*wild* if $|\#(a,s) - \#(b,s)| > \gamma$, where γ is a prespecified parameter, and $\#(a,s)$ is the number of appearances of a in s. For a string $w \in \{a,b\}^*$ and parameters γ and τ, breaking it into strings $s_1, s_2, \ldots, s_\ell$ is a (γ,τ)-***breakup*** of w iff:

(I) For all i, $s_i \in \{a,b\}^*$.
(II) For all i, s_i is γ-wild.
(III) $w = s_1 s_2 \ldots s_\ell$ (that is, the concatenation of $s_1, s_2, \ldots, s_\ell$ is w),
(IV) $\ell \leq \tau$.

Given as input a string $w \in \{a,b\}^*$ of length m, and parameters γ and τ, an algorithm can decide if there is a (γ,τ)-breakup of w in (faster is better):

(A) $O(m^2\tau)$ time.
(B) $O(m^4)$.
(C) $O(m^2\gamma)$ time.
(D) $O(m^3\gamma)$ time.
(E) $O(m^3\tau)$ time.

34. (3 points) You are given two algorithms B_Y and B_N. Both algorithms read an undirected graph G and a number k. If G has a vertex cover of size $\leq k$, then B_Y would stop (in polynomial time!) and output YES (if there is no such vertex cover then B_Y might run forever). Similarly, if G does not have a vertex cover of size $\leq k$, then the algorithm B_N would stop in polynomial time, and output NO (if there is such a vertex cover then B_N might run forever).
In such a scenario:

(A) At least two of the other answers are correct.
(B) This would imply that $P \neq NP$.
(C) One can in polynomial time output if G has a an vertex cover of size $\leq k$ or not.
(D) This would imply that $P = NP$.
(E) Impossible since $P \neq NP$.

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35. (3 points) Let $L_1 \subseteq \Sigma^*$ be a context-free language, and let $L_2 \subseteq \Sigma^*$ be regular. Then the language $L_1 \cap L_2$ is always context-free.

(A) False if the languages L_1 and L_2 are decidable.
(B) False.
(C) None of the other answers.
(D) True if the languages L_1 and L_2 are decidable.
(E) True.

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36. (3 points) Given k sorted arrays $A_1, A_2, \ldots, A_k$ with a total of n numbers stored in them (all numbers are distinct). Given a number x, one can compute the smallest number y, in these arrays, that is larger than x, in (faster is better)

(A) $O(n)$ time.
(B) $O(k \log n)$ time.
(C) $O(n \log n)$ time.
(D) $O(n^2)$ time.
(E) $O(nk)$ time.