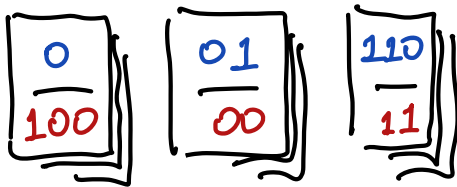NP·hard = no fast algorithm   3SAT, Coloring, TSP,...
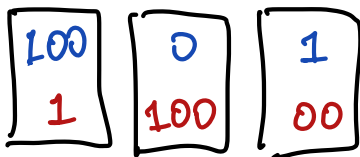
Undecidable = no algorithm at all

| $\frac{0}{100}$ | $\frac{01}{00}$ | $\frac{110}{11}$ |

Post Correspondence
Problem

| $\frac{110}{11}$ | $\frac{01}{00}$ | $\frac{110}{11}$ | $\frac{0}{100}$ |
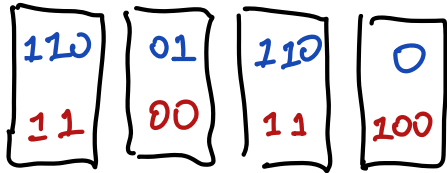
110011100

| $\frac{100}{1}$ | $\frac{0}{100}$ | $\frac{1}{00}$ |

No solution $\longrightarrow$ answer = FALSE

| $\frac{0}{001}$ | $\frac{001}{1}$ | $\frac{1}{0}$ |
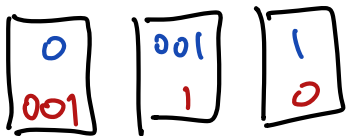
Shortest solution has length 75

COLLATZ(n):
  while n > 1
    if n is even
      n ← n/2
    else
      n ← 3n+1
  return TRUE

7 → 22 → 11 → 34 → 17 → · ···

FRACTRAN
John Conway

$\left[\ \frac{3}{5} \quad \frac{7}{2} \quad \frac{25}{7} \quad 2\ \right]$
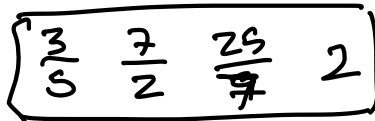
Undecidable

Formally: Turing Machine
Informally: Python

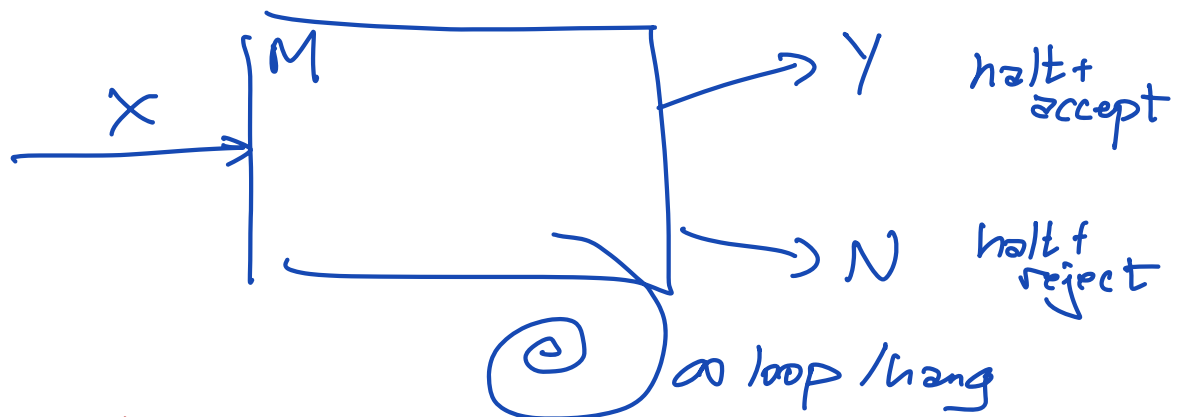$\Big\langle$ Code = executable code / machine
$\quad$ Description: source

Descriptions are interpretable by _code_

M = machine $\qquad \langle M \rangle$ = description

exec. code $\xleftarrow{\text{compiles}}$ Python source

chip $\longleftarrow$ Verilog

Decision problems — correct answer is Yes or No

$\qquad \qquad \qquad \qquad \qquad$ Y $\quad$ halt + accept

$\qquad \qquad \qquad \qquad \qquad$ N $\quad$ halt + reject

$\qquad \qquad \qquad \qquad \qquad \infty$ loop / hang

input $x \in \Sigma^*$

languages $\Big\langle$

$\text{ACCEPT}(M) = \{ x \in \Sigma^* \mid M \text{ accepts } x \}$

$\text{REJECT}(M) = \qquad \qquad$ rejects

$\text{HANG}(M) = \qquad \qquad$ hangs on

SELF REJECT: Given $\langle M \rangle$, does M reject $\langle M \rangle$?

Suppose program SR decides SELF REJECT

$\qquad \text{ACCEPT}(SR) = \text{SELF REJECT}$
$\qquad \text{REJECT}(SR) = \Sigma^* \setminus \text{SELF REJECT}$
$\qquad \text{Hang}(SR) \subseteq \emptyset$

For _all_ machines M:

$\qquad$ SR accepts $\langle M \rangle \Rightarrow M$ rejects $\langle M \rangle$
$\qquad$ SR rejects $\langle M \rangle \Rightarrow M$ does not reject $\langle M \rangle$

In particular $\quad$ SR accepts $\langle SR \rangle \Rightarrow$ SR rejects $\langle SR \rangle$
$\qquad \qquad \qquad$ SR rejects $\langle SR \rangle \Rightarrow$ SR does not reject $\langle SR \rangle$

Diagonalization

Non-presidents Club can't exist

because every club needs a president.