1950s — perebor — brute force <u>exponential</u>

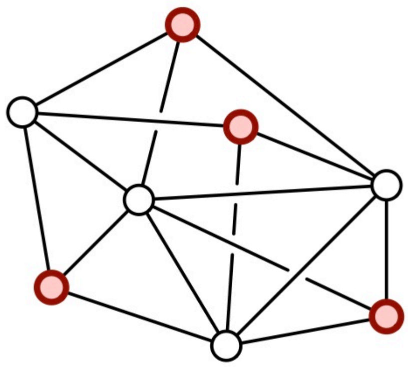
TSP — Find shortest cycle in G visits every vertex

$O(n!) \rightarrow$ Brute Force: Try <u>every</u> permutation of vertices.

SAT — Find inputs that make a boolean formula evaluate to TRUE

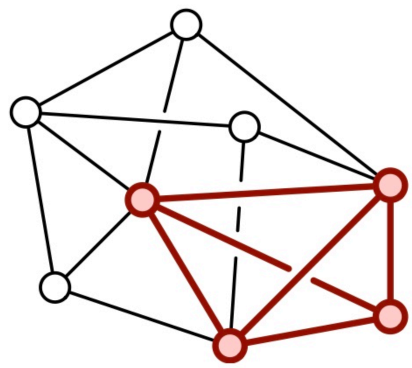
$O(2^n) \rightarrow$ Brute force: Try all possible values for inputs

<u>Reductions</u>: IF X is solvable quickly, then so is Y



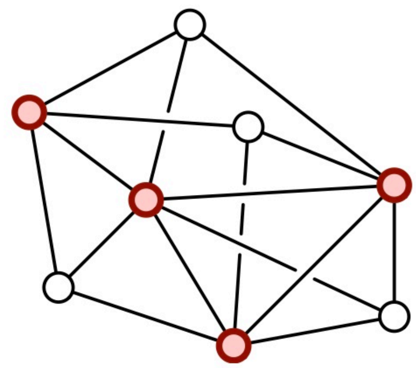
Independent Set

Q: Largest?



Clique

Q: Largest?



Vertex Cover

Q: Smallest?

Max Clique $\longrightarrow$ Max IndSet

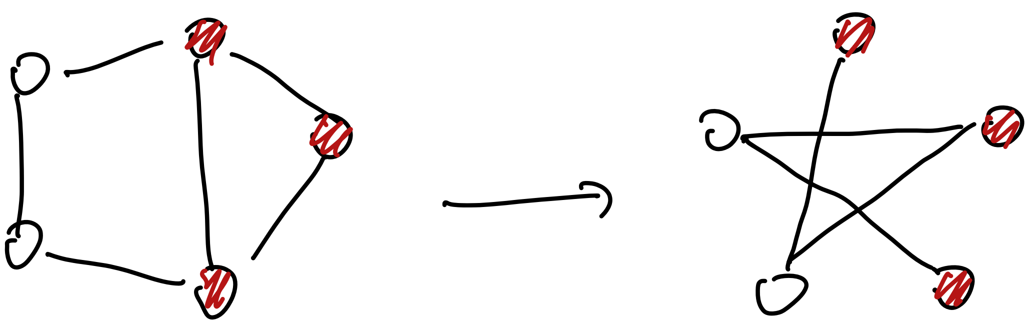
Given a graph $G = (V, E)$

Build a new graph $G' = (V', E')$

$V' = V$

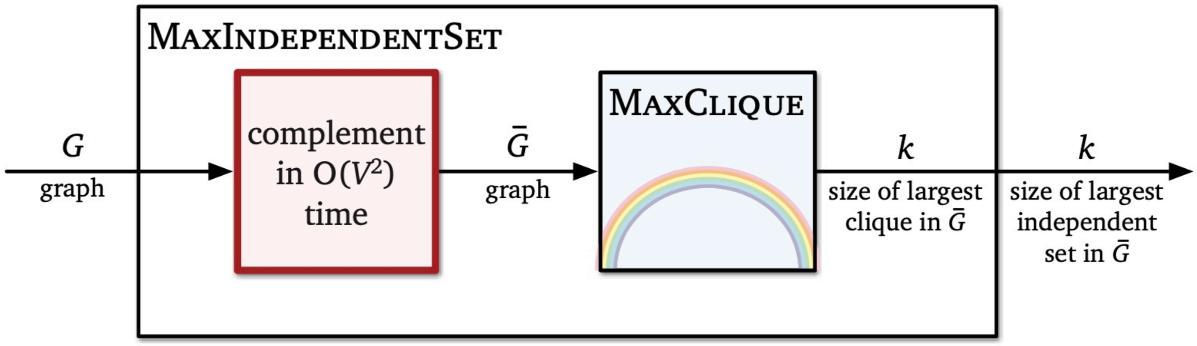
$E' = \{uv \mid uv \notin E\}$

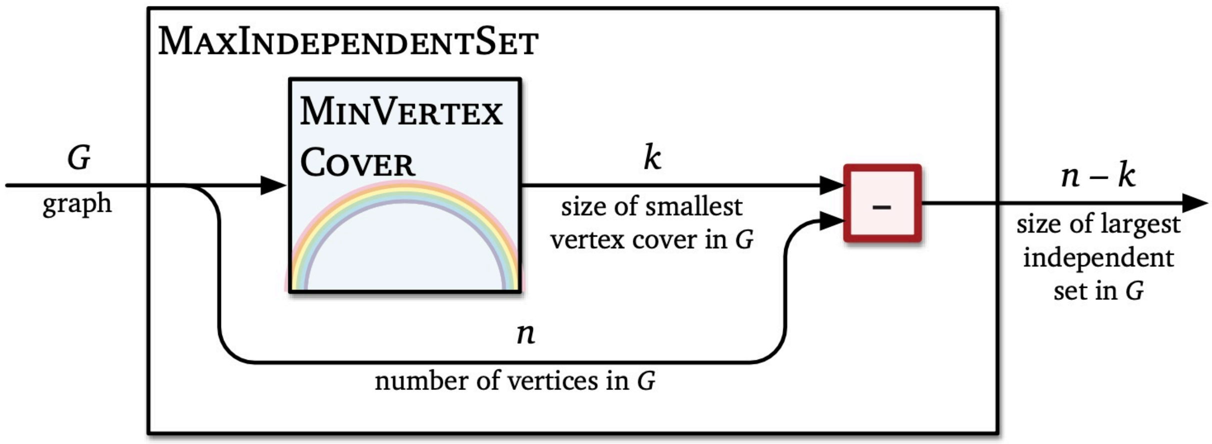
$A \subseteq V$ is a clique in G iff A is independent in G'



Clique $\longrightarrow$ Max Ind Set

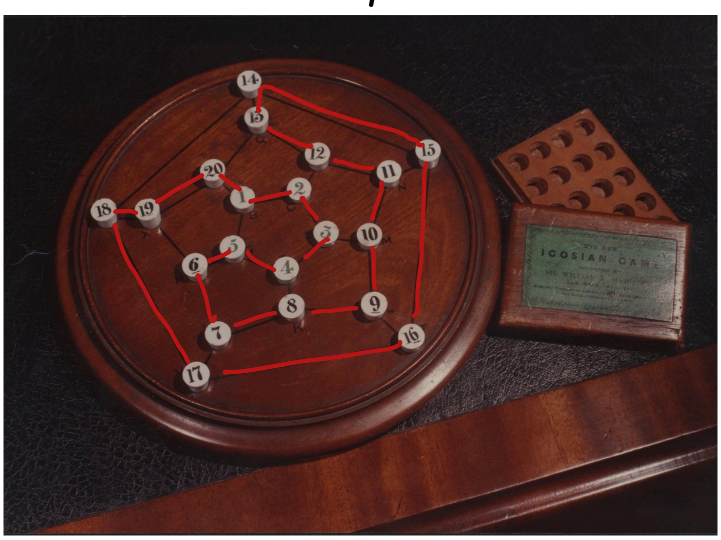
$$T_{Clique}(V) \le O(V^2) + T_{IndSet}(V)$$

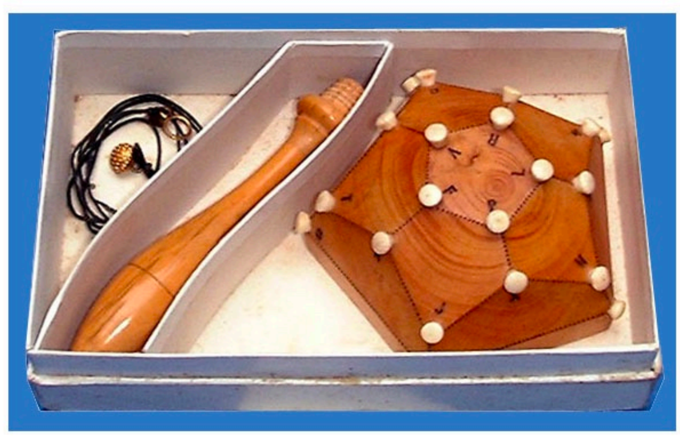




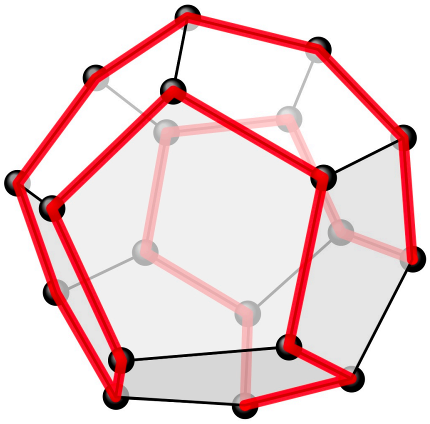

```
MaxIndSet(G):
    n = #V(G)
    return n − MinVertexCover(G)
```

Icosian puzzle





Hamiltonian Cycle
= cycle that visits each vertex exactly once



Given a graph G, does G have a Ham. cycle?

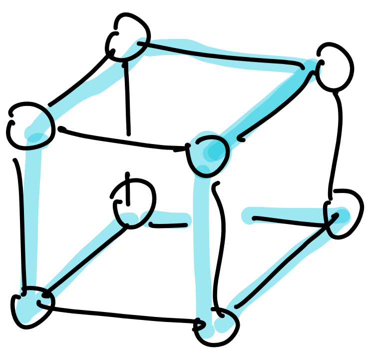
directed — undirected

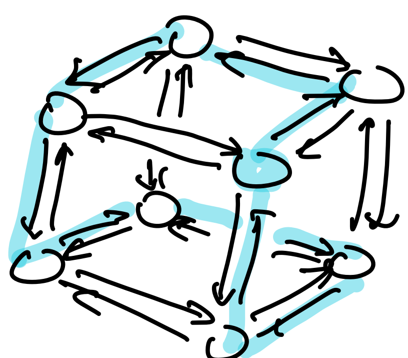
Reduce UndirHamCycle to DirHamCycle

Given undirected $G=(V,E)$

$\downarrow$

Construct directed $G'=(V,E')$
s.t. G has Ham cycle iff G' has Ham cycle.





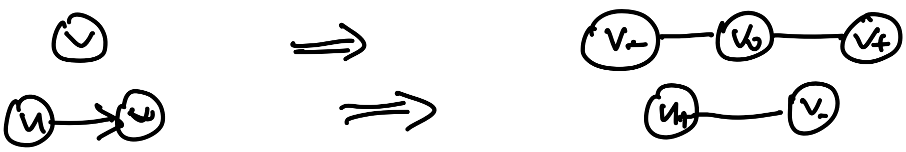
abcdefgh $\overset{\text{if}}{\Longrightarrow}$ $a \to b \to c \to d \to e \to f \to g \to h$

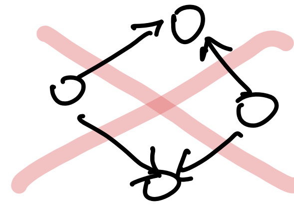
$\underset{\text{F}}{\Longleftarrow}$

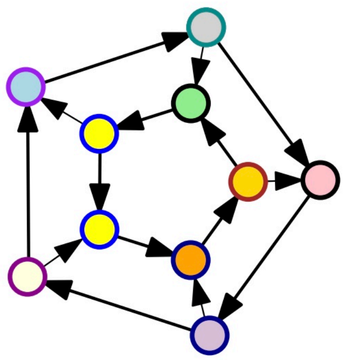
Reduce DirHamCycle to UndirHamCycle

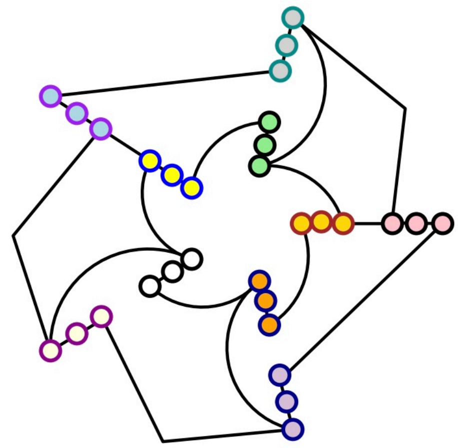
Given dirgraph G $\longrightarrow$ Build undirgraph G'

G has Ham $\rightleftarrows$ G' has Ham.





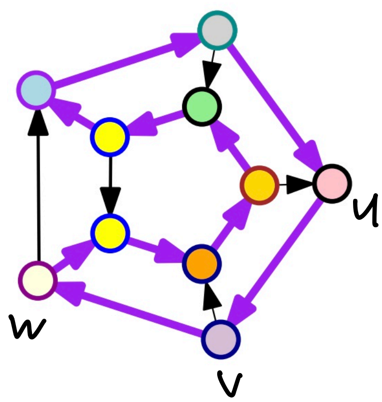


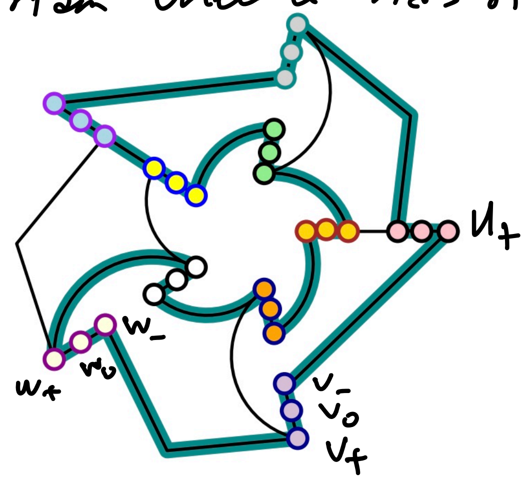


$\#V' = 3V$

$\#E' = 2V + E$

Claim 1: If G has Ham then G' has Ham cycle





Given $u \to v \to w \cdots \to u$ Ham cycle

$\boxed{u_- \to u_0 \to u_+} \to \boxed{v_- \to v_0 \to v_+}$

$\downarrow$

w_-

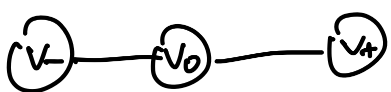
$\vdots$

u_-

$\square$

Claim 2: G' has Ham $\overset{C'}{\Rightarrow}$ G has Ham

C' visits v_0



C' visits $\boxed{v_- \to v_0 \to v_+}$ ← WLOG otherwise reverse C'

or $v_+ \to v_0 \to v_-$

after $v_+ \to w_- \to w_0 \to w_+ \to x_- \to x_0 \to x_+ \to y_-$

C' traverses every edge gadget the same way by induction