Regular languages
- Regular expressions
- DFA
- NFA

Closure properties:

For any reg. langs A and B

$A \cup B$ is regular
$A \cap B$ is regular
$\Sigma^* \setminus A = \overline{A}$ is regular

product construction

$\boxed{(A \cap (B \cup C)) \setminus (B \oplus D)}$

Given DFAs
$M_A = (Q_A, s_A, A_A, \delta_A)$ accepts A
$M_B = (Q_B, s_B, A_B, \delta_B)$ accepts B

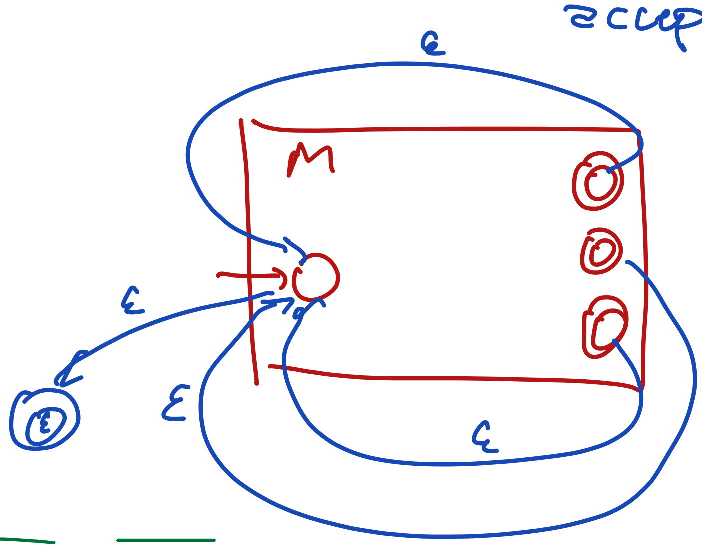
Build DFA $M' = (Q', s', A', \delta')$ accepts $A \cap B$:

$$Q' = Q_A \times Q_B$$
$$s' = (s_A, s_B)$$
$$A' = A_A \times A_B$$
$$\delta((q_A, q_B), a) = (\delta_A(q_A, a), \delta_B(q_B, b))$$

Given any regular language L, prove L^* regular.

Let $M = (Q, s, A, \delta)$ be any DFA accepting L.
We build an NFA with ε-transitions
$M' = (Q', s', A', \delta')$ accepts L^*

To build M', add ε-transitions from accepting states back to s



$Q' = Q \cup \{\varepsilon\}$

$s' = s$

$A' = A \cup \{\varepsilon\}$

$\delta'(q,a) = \delta(q,a)$

$\delta'(q,\varepsilon) = \begin{cases} \{s\} & \text{if } q \in A \\ \emptyset & \text{otherwise} \end{cases}$

$= \{\varepsilon\}$ if $q = s$

Q' = Q u {s', t'}

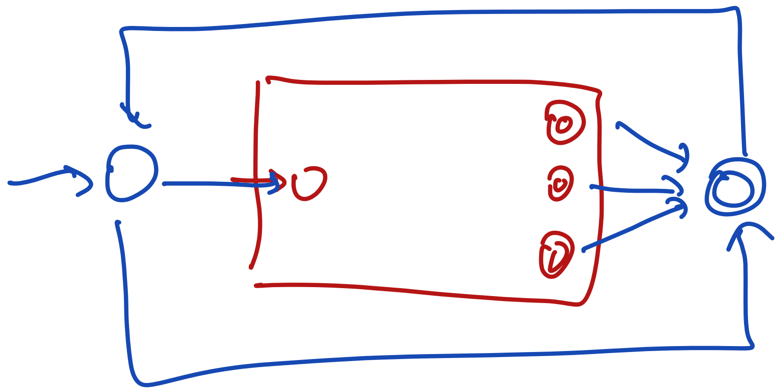
s' = new state

A' = { t' }

δ'(q, a) = { δ(q,a) }   for all q in Q

δ'(s',ε) = { s }

δ'(q, ε} = { t } if q in A else Ø



$$\text{Flip}(w) = \begin{cases} \varepsilon & w = \varepsilon \\ 1 \cdot \text{Flip}(x) & w = 0x \\ 0 \cdot \text{Flip}(x) & w = 1x \end{cases}$$

$\text{FLIP}(0^*1^*) = 1^*0^*$

$\text{FLIP}(L) = \{\text{flip}(w) \mid w \in L\}$

Prove: if L is regular then FLIP(L) is regular.

Proof by induction:

Let R be any reg. exp. such that $L(R) = L$.

$R = \emptyset \Rightarrow F = \emptyset$

$R = w \Rightarrow F = \text{flip}(w)$

$R = A + B \Rightarrow F = \text{FLIP}(A) + \text{FLIP}(B)$

$R = A \cdot B \Rightarrow F = \text{FLIP}(A) \cdot \text{FLIB}(B)$

$R = A^* \Rightarrow F = (\text{FLIP}(A))^*$

Let $M=(Q,s,A,\delta)$ be any DFA accepts L

We build DFA $M'=(Q',s',A',\delta')$ accepts $\text{FLIP}(L)$:

$$Q'=Q$$
$$s'=s$$
$$A'=A$$
$$\delta'(q,0)=\delta(q,1)$$
$$\delta'(q,1)=\delta(q,0)$$

$$\text{rev}(w)=\begin{cases}\varepsilon & \text{if } w=\varepsilon\\ \text{rev}(x)\cdot a & \text{if } w=ax\end{cases}$$

$$\text{REV}(L)=\{\text{rev}(w)\mid w\in L\}$$

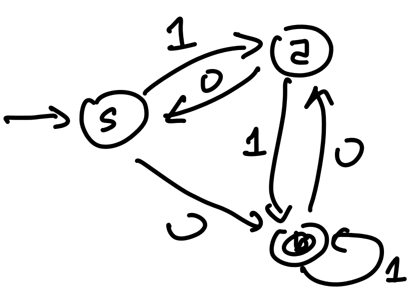
$\text{REV}(0^*1^*)=1^*0^*$

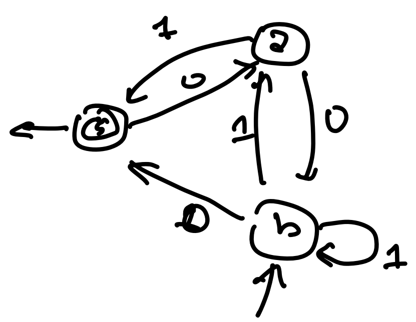
$\text{REV}((011)^*)=(110)^*$

For any reg. lang L, prove $\text{REV}(L)$ is also regular.

Let $M=(Q,s,A,\delta)$ be any DFA accepts L

We build NFA $M'=(Q',S',A',\delta')$ accepts $\text{REV}(L)$:
w/multiple start states





Reverse all the transitions!

$s \xleftarrow{1} a \xleftarrow{0} s \xleftarrow{1} a \xleftarrow{1} b \xleftarrow{1} b$

$s \xrightarrow{1} a \xrightarrow{0} s \xrightarrow{1} a \xrightarrow{1} b \xrightarrow{1} b$

$$Q'=Q$$
$$S'=A$$
$$A'=\{s\}$$
$$\delta'(q,a)=\{p\mid \delta(p,a)=q\}$$

"~~$\delta^{-1}(q,a)$~~"

$\text{PALIN}(L) = \{ w \mid w \cdot \text{rev}(w) \in L \}$ $\quad \cancel{\neq L \cdot \text{REV}(L)}$

$01101 \vdots 10110 \in L \implies 01101 \in \text{PALIN}(L)$

Let $M = (Q, s, A, \delta)$ be any DFA accepting L

Build NFA (mult. starts) $M' = (Q', S', A', \delta')$ accepting $\text{PALIN}(L)$:

"Product of M and M^R"

$$Q' = Q \times Q$$

$$S' = \{ (s, s') \mid s' \in A \}$$

$$A' = \{ (q, q) \mid q \in Q \}$$

$$\delta'((q, r), a) = \{ (\delta(q, a), p) \mid \delta(p, a) = r \}$$

$$s \xrightarrow[M]{0\ 1\ 1\ 0\ 1} q \xleftarrow[M^R]{1\ 0\ 1\ 1\ 0} r \in A$$

$$s \xrightarrow{0\ 1\ 1\ 0\ 1\ \ 1\ 0\ 1\ 1\ 0} r$$