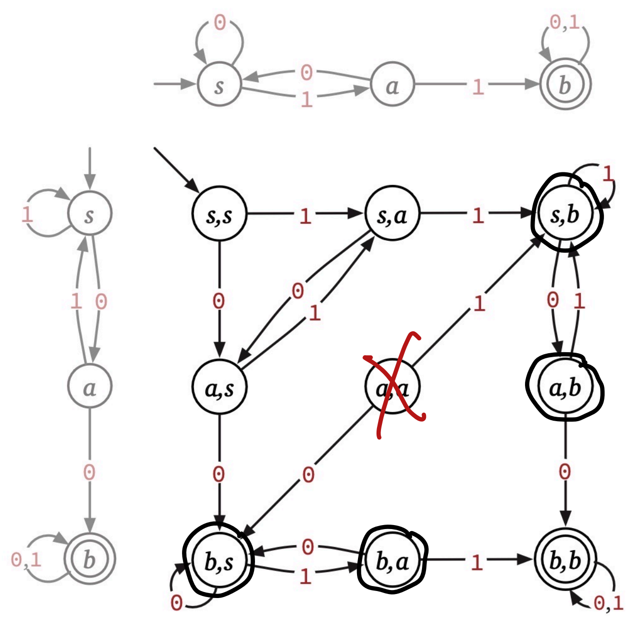
HW 1.1 graded → Gradescope → regrade requests

"You were too lenient"

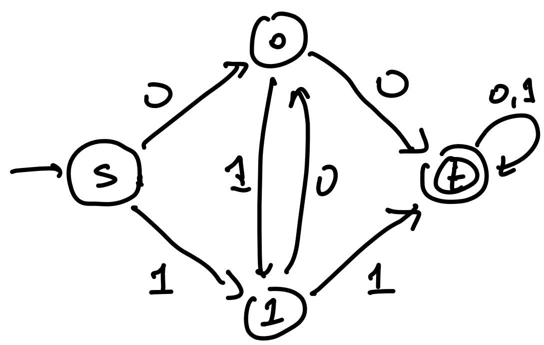
"There is a major bug in solutions"

GPS2 due 9pm
HW2 due tomorrow 9pm

GP3 due Mon 9pm
HW3 due next Tue 9pm



Building a DFA for the language of strings containing ~~both~~ 00 ~~and~~ 11.
~~either~~ or



Is this the smallest DFA for $(0+1)^*(00+11)(0+1)^*$?

YES

Two states q and q' are distnishable iff there is a string w s.t. $\delta^*(q,w)$ or $\delta^*(q',w)$ is accepting but not both.

$\iff$ Either <u>one</u> of q and q' is accepting or there is a symbol a s.t. $\delta(q,a)$ and $\delta(q',a)$ are distinguishable.

s, t dist by ε	s, 0 dist by 0
0, t " " ε	s, 1 dist by 1
1, t " " ε	0, 1 dist by 0 or 1

Strings x and y are distinguishable wrt L
iff there is a string z s.t.
$xz \in L$ or $yz \in L$ but not both.

Set F of strings $\{\varepsilon, 0, 1, 00\}$ $\quad L = (0+1)^*(00+11)(0+1)^*$
is a Fooling-set for L

$\varepsilon, 00$ are distinguished by ε $\quad [\varepsilon \notin L, 00 \in L]$
$0, 00$ $\quad \varepsilon$
$1, 00$ $\quad \varepsilon$
$\varepsilon, 0$ $\quad 0$
$\varepsilon, 1$ $\quad 1$
$0, 1$ $\quad 1$

$\Longrightarrow$ Any DFA for L must have ≥ 4 states.

$$\boxed{\text{Min \# states in DFA} = \text{Max \# strings in a fooling set}}$$

$L = \{0^n 1^n \mid n \geq 0\} \neq 0^* 1^*$
$= \{\varepsilon, 01, 0011, 000111, \ldots\}$

$F = \{\varepsilon, 0, 00, 000, 0000, 00000, 0^6, 0^7, \ldots\}$

ε and 0 dist by 1
ε and 00 " " 11
0 and 00 " " 11
0 and 000 $\quad 111$ $\quad 000111111$

0^i and 0^j dist by 1^i $\quad i \neq j$
$0^i 1^i \in L$ but $0^i 1^j \notin L$

<u>L is not regular!</u>

$L = \{0^n 1^n \mid n \geq 0\}$ is not regular.

Proof:

Let $F = 0^* = \{0^n \mid n \geq 0\}$

Let x and y be any two strings in F

Then $x = 0^i$ and $y = 0^j$ for some $i \neq j$

Let $z = 1^j$

- $xz = 0^i 1^j \notin L$ because $i \neq j$
- $yz = 0^j 1^j \in L$

So z distinguishes x and y

So F is a fooling set for L

F is infinite, so L cannot be regular

L = palindromes $= \{w \in \Sigma^* \mid w = rev(w)\}$

001010100

Theorem. L is not regular.

Proof: Consider the set $F = \{0^i 1 \mid i \geq 0\}$

Let x and y be arbitrary distinct strings in F

Then $x = 0^i 1$ and $y = 0^j 1$ for some $i \neq j$

Let $z = 0^i$

- Then $xz = 0^i 1 0^i \in L$
- But $yz = 0^j 1 0^i \notin L$ because $i \neq j$

So z is dist. suffix for x and y

So F is a fooling set for L

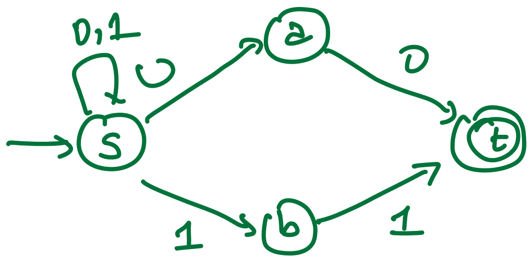
F is infinite $\Longrightarrow$ L is not regular $\square$

Kleene's Theorem

regular $\Longleftrightarrow$ automatic

$\text{DFA} \rightleftarrows \text{NFA} \rightleftarrows \text{reg.expr.}$

Nondeterministic FA



$s \xrightarrow{0} \{a,s\} \xrightarrow{1} \{b,s\} \xrightarrow{1} \{s,b,t\} \xrightarrow{0} \{a,s\}$

Q — states

s — start state

A — accepting states

$\delta : Q \times \Sigma \to 2^Q$ ← subsets of Q powerset of Q $\mathcal{P}(Q)$

$\delta^* : Q \times \Sigma^* \to 2^Q$

$$\delta^*(q,w) = \begin{cases} \{q\} & w = \varepsilon \\ \bigcup_{q' \in \delta(q,a)} \delta^*(q',x) & w = ax \end{cases}$$

M accepts w iff $\delta^*(s,w) \cap A \neq \emptyset$