You have 120 minutes to answer five questions.

Write your answers in the separate answer booklet.

Please return this question sheet and your cheat sheet with your answers.

1. For each of the following languages L over the alphabet $\Sigma = \{0, 1\}$, describe a DFA that accepts L **and** give a regular expression that represents L. You do not need to justify your answers.

   (a) All strings in which the number of runs is divisible by 3. (Recall that a *run* is a maximal non-empty substring where all symbols are equal.)

   (b) All strings that do not contain the substring 0110.

2. Let take2skip2(w) be a function takes an input string w and returns the subsequence of symbols at positions $1, 2, 5, 6, 9, 10, \dots 4i+1, 4i+2, \dots$ in w. In other words, take2skip2(w) takes the first two symbols of w, skip the next two, takes the next two, skips the next two, and so on. For example:

$$\begin{aligned}\text{take2skip2}(\underline{1}) &= 1\\ \text{take2skip2}(\underline{01}0) &= 01\\ \text{take2skip2}(\underline{01}00\underline{11}11\underline{00}01\underline{1}) &= 0111001\end{aligned}$$

   Let L be an arbitrary regular language.

   (a) ***Prove*** that the language $\{w \in \Sigma^* \mid \text{take2skip2}(w) \in L\}$ is regular.

   (b) ***Prove*** that the language $\{\text{take2skip2}(w) \mid w \in L\}$ is regular.

3. Consider the following recursive function censor, which deletes all 1s in its input string.

$$\text{censor}(w) := \begin{cases} \varepsilon & \text{if } w = \varepsilon \\ \text{censor}(x) & \text{if } w = 0 \cdot x \text{ for some string } x \\ 1 \cdot \text{censor}(x) & \text{if } w = 1 \cdot x \text{ for some string } x \end{cases}$$

   (a) ***Prove*** that $|\text{censor}(w)| \le |w|$ for all strings w.

   (b) ***Prove*** that $\text{censor}(\text{censor}(w)) = \text{censor}(w)$ for all strings w.

   As usual, you can assume any result proved in class, in the lecture notes, in labs, or in homework solutions.

4. Consider the language $L = \left\{ 0^a 1^b \;\middle|\; a > 2b \text{ or } 2a < b \right\}$

   (a) ***Prove*** that L is *not* a regular language.

   (b) Describe a context-free grammar for L.

5. For each statement below, check "True" if the statement is always true and check "False" otherwise, and give a brief (one short sentence) explanation of your answer. Read these statements very carefully—small details matter!

   (a) For every language L, the language L^* is infinite.

   (b) If a language L is finite, the complement of L is context-free.

   (c) The language $\{0^{374n} \mid n \geq 374\}$ is regular.

   (d) The language $\left\{ wxw^R \;\middle|\; w, x \in \Sigma^* \right\}$ is regular.

   (e) The context-free grammar $S \to 0S1S \mid S1S0 \mid \varepsilon$ generates the set of all binary strings with the same number of 0s and 1s.

   (f) Every regular language is recognized by a DFA with at least 374 states.

   (g) If the languages L and L' are regular, their intersection $L \cap L'$ is also regular.

   (h) If a language has an infinite fooling set, then it is context-free.

   (i) Let M be a **DFA** over the alphabet Σ. Let M' be identical to M, except that accepting states in M are non-accepting in M' and vice versa. Each string in Σ^* is accepted by exactly one of M and M'.

   (j) Let M be an **NFA** over the alphabet Σ. Let M' be identical to M, except that accepting states in M are non-accepting in M' and vice versa. Each string in Σ^* is accepted by exactly one of M and M'.