Wednesday, February 16, 2022 6:12 PM

Midterm 1 Review

Feb 21, Monday 7-9 pm

Cover DFA, regular, NFA, CFG (up to lec 8)

But no TM.

#11. All strings containing at least two 1s and at least one 0.

$0\sim 1 \sim 1 \sim$ OR $\underbrace{1 \dots 1}_{\text{only 1 one}}\, 0 \sim 1 \sim$ OR

$1 \cdots 1 0 \cdots$

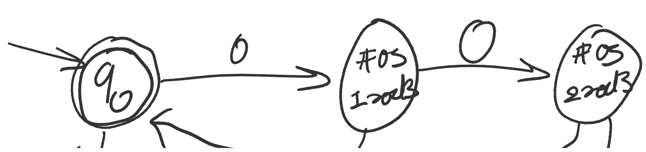
0^*

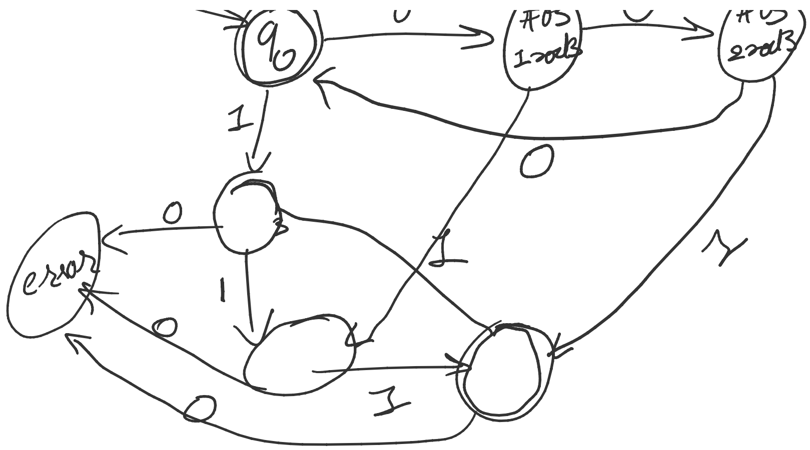
$$\underline{0 0^* 1 (0+1)^* 1 (0+1)^*} + 1 1^* 0 0^* 1 (0+1)^* + 1 1 1^* 0 (0+1)^*$$

#26. The set of all strings in 0*1* whose length is divisible by 3.

ε, 000111, 011, 000001111

$$\underbrace{(000)^* \cdot (111)^*}_{\text{\#0s is 0 mod 3, \#1s " "}} + \underbrace{(000)^* 011 (111)^*}_{\text{\#0s is 1 mod 3, \#1s is 2 mod 3}} + \underbrace{(000)^* 001 (111)^*}_{\text{\#0s is 2 mod 3, \#1s is 1 mod 3}}$$





#35. L = {0^Fn | n>=0}, where Fn is the nth Fibonacci number, defined recursively as follows:

F0=0, F1=1, Fn=F(n-1) + F(n-2)

$$L = \{0^{F_n} \mid n \geq 0\}$$

$$F_0 = 0$$
$$F_1 = 1$$
$$F_n = F_{(n-1)} + F_{(n-2)} \quad \forall n > 1$$

Fooling set

$$F = \{0^{F_i} \mid i \geq 3\}$$

Given any two strings $x, y \in F$ $\quad x \neq y$

$x = 0^{F_i}$, $y = 0^{F_j}$ $\quad$ wlog $j > i$

Pick $w = 0^{F_{(i-1)}}$

s.t. $xw = 0^{F_i} \cdot 0^{F_{(i-1)}} = 0^{F_i + F_{(i-1)}} = 0^{F_{(i+1)}} \in L$

$yw = 0^{F_j} \cdot 0^{F_{(i-1)}} = 0^{F_j + F_{(i-1)}}$

$$F_j < F_j + F_{(i-1)} \underset{(\because j > i)}{\leq} F_j + F_{(j-1)} = F_{(j+1)}$$

$i=2 \quad j=3$

$F_0=0 \quad \underbrace{\underline{F_1=1} \quad \underline{F_2=1}} \quad F_3=2 \quad F_4=3 \ldots$

#45. L={www | w \in {0,1}*}

$000\,000$

$F=\{0^i 1 0^i 1 \mid i \geq 1\}$

$x, y \in F, \quad x \neq y$

$y = 0^j 1 0^j 1 \quad j > i$

Given, $x = 0^i 1 0^i 1$

Pick $z = 0^i 1$

$xz = \underbrace{0^i 1}_{w} \underbrace{0^i 1}_{w} \underbrace{0^i 1}_{w} \in L$

$yz = 0^j 1 0^j 1 0^i \underline{1} \notin L$

$|yz| = 3l$

$\Rightarrow l < j+1$

ends at 0 $\quad \underbrace{00\cdots 0}_{l}\underbrace{010\cdots 0}_{l}\underbrace{\cdots 010\,01}_{l}$ ends at 1

$L = \{wxw \mid \underline{x}, \underline{w} \in \{0,1\}^*\} = \{0,1\}^*$, $w = \varepsilon$

Regular. $(\because w = \varepsilon)$

#48. The set of all strings in {0, 1}* such that in every non-empty prefix, the number of 0s is greater than the number of 1s.

60. All strings that satisfy ~~all~~ any one of the following:

60.A. the number of 0s is even

60.B. the number of 1s is divisible by 3

60.C. the total length is divisible by 5

DFA $M = (Q, \underbrace{\Sigma}_{\{0,1\}}, q_0, \delta, A)$ → A'

where

$Q = \{ (\underbrace{i}_{\#0\text{'s mod } 2}, \underbrace{j}_{\#1\text{'s mod } 3}, \underbrace{k}_{\text{length mod } 5}) \mid i \in \{0, 1\},\ j \in \{0,1,2\},\ k \in \{0,1,2,3,4\} \}$

$q_0 = (0,0,0)$

$\delta((i,j,k), 0) = ((i+1) \bmod 2,\ j,\ (k+1) \bmod 5)$

$\delta((i,j,k), 1) = (i,\ (j+1) \bmod 3,\ (k+1) \bmod 5)$

$A = \{ (0,0,0) \}$

$A' = \{ (0,j,k) \mid j \in \{0,1,2\},\ k \in \{0,1,2,3,4\} \} \cup$

$\{ (i,0,k) \mid \ldots \} \cup$

$\{ (i,j,0) \mid \ldots \}$

89. $\{ w\#0^{\overbrace{\#(0,w)}^{\#0\text{s in } w}} \mid w \in \{0,1\}^* \}$ (write CFG)

$11011 \# 0$ $\quad$ $01011 \# 00$

$1001011 \# 000$

$S \to 0S0 \mid 1S \mid \#$

#77. $Even(L) = \{ evens(w) \mid w \in L \}$

If L is regular then $Even(L)$ is regular.

$odds(w) = \varepsilon$ if $w = \varepsilon$

$= a\, evens(x)$ if $w = ax$

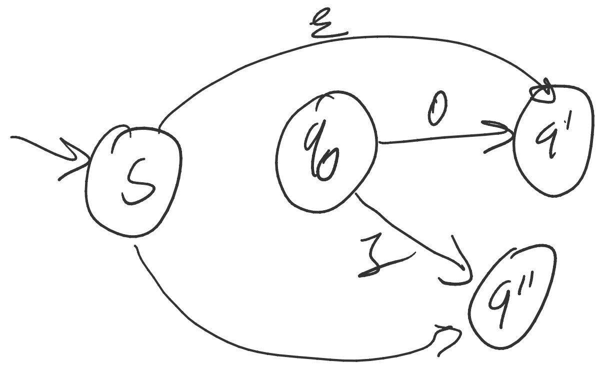
$evens(w) = \varepsilon$ if $w = \varepsilon$

$= odds(x)$ if $ax = w$, $a \in \Sigma$, $x \in \Sigma^*$

$evens(001010) = 000$ $\quad$ $odds(001010) = 011$

Let $M = (Q, \Sigma, q_0, \delta, A)$ be DFA for L.

Construct NFA, M' for $EVEN(L)$ $M' = (Q', \Sigma, s, \delta', A')$



skip take

$$Q' = Q \times \{\overset{\text{skip}}{\varepsilon}, \overset{\text{take}}{t}\}$$

$$s = (q_0, \varepsilon)$$

$$\delta'((q, \overset{\text{skip}}{\varepsilon}), \varepsilon) = \{(\delta(q,0), t), (\delta(q,1), t)\}$$

$$\delta'((q, \underset{\text{take}}{t}), 0) = \{(\delta(q,0), \varepsilon)\}$$

$$\delta'((q,t), 1) = \{(\delta(q,1), \varepsilon\}$$

$$A' = \underbrace{\{(q, \varepsilon) \mid q \in A\}}_{\text{odd length}} \cup \underbrace{\{(q,t) \mid q \in A\}}_{\text{even length}}.$$

$\text{even}(010) = 1$ — $\varepsilon\ t\ \varepsilon$

$\text{even}(0011)$ — $\varepsilon\ t\ \varepsilon\ t\ \varepsilon$